The nuclear spin isospin response
The $({\vec d}, 2p)$ reaction at intermediate energies

Licentiatafhandling i eksperimentel kernefysik ved Niels Bohr Institutet, Københavns Universitet.

København, den 20. december 1990

Thomas Sams

Licentiatforelæsning afholdt den 27. februar 1991 på NBI.
ISBN 87-987519-0-5
http://www.nbi.dk/~sams
## Contents

1 Prologue 1

2 Experiment and basic ideas 4
   2.1 Plane wave description 6
   2.2 The plane wave form factor 6
   2.3 Reference targets 9

3 Spin structure of the $\Delta$ excitation 11
   3.1 One pion exchange and Poor Mans Absorption model 11
   3.1.1 Poor mans absorption model 12
   3.1.2 On shell approximation 13
   3.1.3 Comparison with experiments 13
   3.2 The nuclear response in the $\Delta$ region 15

4 The quasifree region 24
   4.1 Distortion prescription when omitting spin dependence 24
   4.2 Including spin 26
      4.2.1 Driving force 26
      4.2.2 Probe wave functions 28
      4.2.3 Scattering matrix including spin dependence 30
   4.3 Eikonal absorption limit of Glauber theory 30
      4.3.1 Physical interpretation of the probe weight function 33
      4.3.2 Symmetries of distortion prescription 34
      4.3.3 The slope of the polarization response 35
   4.4 Response function 36
      4.4.1 Local density approximation 37
      4.4.2 Response on reference targets 41
   4.5 Experimental results 42

5 Epilogue 54

6 Thanks 54

A Description of polarized particles 56
   A.1 The spin density operator and the analyzing powers 56
   A.2 Helicity representation 58
   A.3 Transformations used in calculations of $p(\vec{d}, 2p)\Delta^0$ 60

B Calculation of deuteron response function 64

C Pion exchange 65
The nuclear spin isospin response
The \((d, 2p)\) reaction at intermediate energies

Thomas Sams

Niels Bohr Institutet, DK-2100 København, Danmark
Laboratoire National Saturne, F-91191 Gif-sur-Yvette Cedex, France

1 Prologue

The spin isospin response of nuclei is the subject of this work. The charge exchange reaction \((d, 2p)\) with polarized deuteron beam at typically 1 GeV bombarding energy per nucleon is the probe with which we aim at measuring the nuclear response. The \((d, 2p[1S_0])\) reaction provides information equivalent to that of the \((n', p')\) reaction without measuring the polarization of the outgoing particle [1, 2, 3, 4]. Figure 1 illustrates the relationship between \((d, 2p)\) and \((n, p)\).

\[
|2p[1S_0]\rangle = \begin{array}{c}
|d M\rangle \\
p \\
\end{array} \times \begin{cases} 
S^+(t), & M = 0 \\
S^-(t), & M = \pm 1 
\end{cases}
\]

Figure 1: Direct amplitude for the \((d, 2p)\) reaction in plane wave impulse approximation. The amplitude for the \(n \rightarrow p\) transition is simply multiplied by the form factor for the transition in the probe. Quantizing along the momentum transfer in the Breit frame the \((d, 2p)\) form factor is diagonal in the spin \(M\) of the deuteron.

The isospin channel of the nucleon-nucleon interaction is highly spin dependent. When the momentum transfer is larger than \(m_\sigma\) the spin longitudinal and the spin transverse channel even have opposite sign. Similar contrast between the longitudinal and transverse spin channel is expected for the residual interaction in an excited nucleus. This could generate differences in the nuclear response in the two channels. The possible difference between the spin longitudinal and the spin transverse response is the subject of the present work.

The strong spin dependence of the interaction is due to the pseudo-scalar nature of the one pion exchange

\[
\mathcal{M} = - \left( \frac{f_\pi}{m_\pi} \right)^2 \left(2m_\pi\right)^2 \frac{(\sigma^1 \cdot q)(\sigma^2 \cdot q)}{m_\pi^2 + q^2} \tau^1 \cdot \tau^2
\]

(1)

here given in the non relativistic limit. The OPE is purely spin longitudinal: the Pauli operator \(\sigma\) acting in the spin space of the nucleon acts “along” the momentum transfer \(q\). However,
even the large spin transverse components of the isospin channel are determined by the pion exchange mechanism: they result from a screening of the OPE at small impact parameters. This postulate, concerning the structure of the isospin channel of interacting hadrons, is found to hold for the two prototype reactions of the isospin channel, the excitation of the $\Delta$ (1232 MeV) isobar $N + N \rightarrow N + \Delta$ and the elementary charge exchange reaction $n + p \rightarrow p + n$. It is most important for the study of the nuclear spin-isospin response, since it means that the spin isospin channel of the nuclear response is determined by the structure of the pion-nucleon vertex.

The experimental study of the quasi elastic excitation of nuclei with the $(^3\text{He}, t)$ reaction results in a dispersion relation deviating substantially from the free

$$\omega_{\text{lab}} = \frac{-t}{2m_N} \text{N.R.} \frac{q^2}{2m_N}$$

(2)

At large momentum transfers, of order $2 - 3p_F$, the quasi elastic peak is shifted downwards to about half of this value, whereas it qualitatively agrees with the value given by (2) at momentum transfers similar to the Fermi momentum. Also with the $(d, 2p)$ probe this behavior is seen. One major reason for studying the spin isospin excitation of nuclei in selected spin channels is to determine whether the altered "dispersion" relation reflects a polarization of the nuclear medium. Notably, the $(p, p')$ and $(e, e')$ reactions follow the free dispersion relation, though for the latter shifted by a constant, and the $(p, n)$ seems to fall closer the free than to that observed in $(d, 2p)$ [5].

Only little attention has been put to the properties of the composite probes, i.e. the question whether the DWBA approach is appropriate to describe the penetration of the probe into the nucleus. Some consideration will be devoted to this question, in particular the question whether the differences between results obtained with different probes is due to inelastic rescattering effects. Though, it will not be answered in terms of a calculation.

The study of the $\Delta$ resonance in a nuclear environment with the $(^3\text{He}, t)$ probe shows a large downwards shift $\sim 70$ MeV of the excitation energy of the $\Delta$ resonance when compared to the free excitation of the $\Delta$ [6]. This has led to the hypothesis that part of the shift could signal effects of a medium polarization in the spin-isospin channel [7]. A calculation of the response of the nuclear medium in the random phase approximation suggests that the spin longitudinal response be shifted downwards (softened) and enhanced, and the spin transverse be hardened and quenched.

Since then much work has been done in the attempt to reach a more detailed understanding of the physical content of the shift [8, 9, 10]. The effects of the finiteness of the nucleus and the fact that densities in the regions probed by hadronic probes are lower than the central densities have been evaluated separately, both lead to a weakening of the signal. Even though the improved calculations indicate a much weaker experimental signal than that originally suggested, the lesson is still the same: the spin longitudinal response should be softened and enhanced, and the spin transverse hardened and quenched.

As is evident from this there is a call for experiments which can isolate the spin-isospin channel and within this channel distinguish between longitudinal and transverse response. The $(\bar{n}, \bar{p})$ and $(\bar{p}, \bar{n})$ probes at intermediate energies are the prototypes of such reactions. However, they suffer from the necessity of having both polarized beam and measuring the polarization of the outgoing nucleon. The rescattering techniques involved in determining the spin of the
outgoing nucleon lead to low efficiencies, and therefore limited amount of data. This led to the idea of using the high quality tensor polarized deuteron beam at Laboratoire National Saturne to study the nuclear response with the charge exchange probe \((\vec{d}, 2p)\). Concerning spin observables, it provides the same rank of information as could be obtained in an \((\vec{n}, \vec{p})\) experiment. However, the effect of distortion on the spin observables obtained with the composite probe are considerable. This means that we have to learn how to correct for distortion.

We are thus faced with two kinds of data. The \((\vec{p}, \vec{n})\) data which are simpler to analyze theoretically, but limited in systematics, i.e. only few targets and so far only done at one momentum transfer [11]. This data taken at Los Alamos is being analyzed now. On the other hand the \((\vec{d}, 2p)\) data which are much richer in systematics but harder to analyze theoretically.
2 Experiment and basic ideas

The $(\bar{d}, 2p)$ experiments have been performed using the polarized deuteron beam at Laboratoire National Saturne. Since the upgrade with the MIMAS pre-accelerator Saturne delivers up to $10^{11}$ deuterons per burst [12]. Further the ion source delivers an 85% tensor polarized beam which remains practically unchanged during the acceleration [13]. This means that, quantized along the symmetry axis of the synchrotron, the experiments have been performed with tensor polarization $\rho_{20} = 0.60 \pm 0.01$ of the beam to be compared to a maximal theoretical value of $1/\sqrt{2}$. Using this unique beam, the experiments were performed at bombarding energies 1.6 GeV and 2.0 GeV. The results at the two energies are found to be consistent.

![Spectrum of the measured difference in time of arrival of the two protons at the intermediate focal plane.](image)

Figure 2: Spectrum of the measured difference in time of arrival of the two protons at the intermediate focal plane. The true events fall inside the peak representing two protons arriving “simultaneously”. Consecutive break-up of two deuterons gives the flat background. This background is minimized setting a software gate around the peak as indicated. The remaining background can be subtracted by setting a gate of the same size in the flat region and subtracting the events falling in this gate from the events inside the true gate.

The two protons were detected individually in the spectrometer SPES4 [14]. The cuts from the $1.7^\circ \times 2.6^\circ$ collimator and from the spectrometer restricts the relative motion of the $2p$ system to the $^1S_0$ state with high efficiency [1]. This serves to select the spin transfer channel. In order to minimize background from sequential break-up of two deuterons, the two protons were required to arrive in coincidence (≈ 2 ns time window) in plastic scintillators at the intermediate focal plane 16 m after the target. By setting the coincidence window a few ns off, the background could be subtracted. A typical spectrum of the time difference is shown in figure 2.

As reference and test targets we have used liquid H$_2$ and D$_2$ (100 – 300 mg/cm$^2$). The yield
from the "empty" target consisting of Ti and mylar foils (total of 30 mg/cm²) was subtracted. The nuclear targets for which we have performed the most detailed studies were \(^{12}\text{C}\) and \(^{40}\text{Ca}\) foils of thicknesses 50–500 mg/cm².

The spectrograph was run in a mode where it covers ±3.5% around the central momentum setting. In \((d,2p)\), however, the efficiency falls off rather fast when the extreme momenta are approached. In order to have sufficient overlap of field settings only the central ±2.5% were used. Spectra covering energy transfers from 0 to 500 MeV could thus be obtained from 7 magnetic field settings each taking typically 30 minutes. Spectra were recorded at 5 angles allowing the momentum transfer to cover from 0 fm\(^{-1}\) to 2.4 fm\(^{-1}\).

In "popular" terms the tensor polarization response we have been able to measure at Laboratoire National Saturne is

\[
P = \frac{\rho_{20}^{\text{beam}} \sigma(\uparrow) + \sigma(\downarrow) - 2\sigma(0)}{\rho_{20}^{\text{max}} \sigma(\uparrow) + \sigma(\downarrow) + \sigma(0)}
\]  

(3)

to be read as: spin-up + spin-down − 2 times spin-0 divided by the sum. The spin is here quantized along the axis of the synchrotron.

In terms of tensor analyzing powers in a co-ordinate system defined by the reaction, the polarization response may be expressed

\[
P = \rho_{20} \left( \frac{1}{2} T_{20}^{M} + \sqrt{\frac{3}{2}} \langle \cos 2\varphi \rangle T_{22}^{M} \right)
\]  

(4)

The tensor analyzing powers \(T_{\lambda\mu}^{M}\) refer to the Madison frame [15], i.e. Z-axis along the beam and Y-axis along the normal \(\hat{n}\) to the reaction plane \(\mathbf{p}_1 \times \mathbf{p}_3\). \(\varphi\) is the angle between the normal to the reaction plane and the direction of the beam polarization, making the expectation value of \(\cos 2\varphi\) essentially a function of the scattering angle. The formalism used to describe the polarized beam is discussed in appendix A.1. When \(\cos 2\varphi = 1\) the polarization response \(P\) is proportional to the cartesian analyzing power \(A_{YY}\), when \(\cos 2\varphi = -1\) it is proportional to the cartesian analyzing power \(A_{XX}\).

As yet only experiments with the symmetry axis of the beam polarization along the symmetry axis of the synchrotron have been performed. This means that \(\langle \cos 2\varphi \rangle \simeq 1\) at finite scattering angles. Future experiments, in which the deuteron spin is precessed in a superconducting solenoid, will make it possible to measure \(T_{20}^{M}\) and \(T_{22}^{M}\) separately, thus providing further information on the spin structure of the reactions. In particular the combination corresponding to \(\cos 2\varphi = -1\) is of interest, since, in the plane wave description, it gives the best separation between the spin transverse and spin longitudinal reaction mechanism. This is due to the fact that, in quasi elastic kinematics, the momentum transfer is essentially parallel to the \(X\)-axis of the Madison frame, making \(A_{XX}\) the quantity that separates spin transfer according to whether there is spin flip (transverse) or not (spin longitudinal) along this axis. However, due to the smallness of one of the spin transverse amplitudes, \(\varepsilon\) in figure 13, the quantity already measured gives a rather good separation. (See the plane wave discussion below.)
2.1 Plane wave description

Assuming the reaction to be dominated by a simple one step mechanism as illustrated in figure 1, we can measure a combination of spin longitudinal \((\sigma \cdot \hat{q})\) and spin transverse \((\sigma \cdot \hat{n} \text{ and } \sigma \cdot (\hat{n} \times \hat{q}))\) response. At strictly forward scattering in the \(\Delta\) region we can even separate the spin transverse and the spin longitudinal cross section. In a frame following the probe, the Breit frame, the amplitude for the \((n, p)\) transition is written

\[
\mathcal{M} = \alpha + \varepsilon \sigma \cdot (\hat{n} \times \hat{q}) + \beta \sigma \cdot \hat{n} + \delta \sigma \cdot \hat{q}
\]

where the spin dependence of the scattering operator in the probe “vertex” is explicitly specified, whereas the spin dependence in the other “vertex” is included in the operators \(\alpha, \beta, \delta\) and \(\varepsilon\). The operator \(\delta\) is referred to as longitudinal, \(\beta\) and \(\varepsilon\) are called transverse. \(\alpha\) does not contribute in the \((d, 2p)\) reaction where the \(\Delta S = 1\) channel is selected. A direct \(\pi\)-exchange diagram gives rise to a purely longitudinal response.

In terms of the \((n, p)\) cross section

\[
d\sigma_{(n,p)} = |\alpha|^2 + |\beta|^2 + |\varepsilon|^2 + |\delta|^2
\]

\[
|\alpha|^2 = \sum_{m_2 m_4} |\langle m_4 | \alpha | m_2 \rangle|^2 \quad \text{etc.}
\]

where kinematical factors have been omitted, the cross section for the \((d, 2p)\) is

\[
d\sigma_{(d,2p)} = N \frac{1}{3} \left( (|\beta|^2 + |\varepsilon|^2) |S^-(t)|^2 + |\delta|^2 |S^+(t)|^2 \right)
\]

\[
= N \frac{1}{3} \left( d\sigma_{(n,p)}^T |S^-(t)|^2 + d\sigma_{(n,p)}^L |S^+(t)|^2 \right)
\]

\(N\) is an absorption factor, and the superscripts be read as transverse (T) and longitudinal (L). The formfactors \(S^\pm\) describing the transition in the probe are discussed in section 2.2.

In general we have taken the attitude to normalize the measured cross section to the \(p(d, 2p)n\) cross section at \(q = 0.7\, \text{fm}^{-1}\) [1, 4]. This process is then calculated in PWIA as described below to obtain absolute normalizations. The effect of absorption on the proton target is taken into account in the Glauber approach giving rise to an absorption factor is around 0.85 on the proton target. This effect is included in the normalization of the experimental spectra in the quasifree region. In the \(\Delta\) region, where in any case only plane wave calculations are shown, the spectra shown are normalized to the PWIA calculation. In publications we have shown the cross sections normalized to the plane wave calculation [1, 16].

2.2 The plane wave form factor

The plane wave formfactors for the transition from the deuteron to the di-proton \([1S_0]\) state

\[
\langle 2p; k \mid \exp\left(i \frac{q \cdot s}{2}\right) \sigma_{1 M}^t \mid d; 1M \rangle = \begin{cases} 
S^+(k, q) & M = 0 \\
S^-(k, q) & M = \pm 1 
\end{cases}
\]

\[
= \left\{ \begin{array}{ll}
\int u_k(s)j_0\left(\frac{qS}{2}\right)U(s)ds + \sqrt{2} \int u_k(s)j_2\left(\frac{qS}{2}\right)W(s)ds & M = 0 \\
\int u_k(s)j_0\left(\frac{qS}{2}\right)U(s)ds - \frac{1}{\sqrt{2}} \int u_k(s)j_2\left(\frac{qS}{2}\right)W(s)ds & M = \pm 1
\end{array} \right.
\]

(8)
Figure 3: The experimental $k$-distribution at bombarding energy $T_{\text{lab}} = 2$ GeV, 2.3° laboratory scattering angle, and collimator 1.7° × 3.4° is compared with the numerical integration. The calculation with (dash) and without (full) restrictions from the spectrometer SPES4 are shown. (The calculation has been redone with a better knowledge of the beam emittance since that presented in ref. [1].)

where the quantization axis was chosen along the momentum transfer in the Breit frame where $|q| = \sqrt{-t}$ [3, 16].

It has proven possible to describe the transition in the probe $(d, 2p)$ by effective formfactors, $S^+(t)$ and $S^-(t)$. They depend on the four momentum transfer only

$$|S^\pm(t)|^2 = \int_0^\infty \left( \frac{d\sigma}{dk} \right)_{\text{spec coll}} \left| \frac{S^\pm(k,t)}{S^\pm(k,t_0)} \right|^2 dk$$

(9)

with $t_0 = -1$ fm$^{-2}$. In (9) the (typical) theoretical $k$-distribution, represented by $|S^\pm(k,t_0)|^2$, is divided out and replaced by the experimentally allowed distribution as calculated taking into account the cuts from the collimator and the spectrograph SPES4. The choice $t_0 = -1$ fm$^{-2}$ has been checked not to be crucial: the detailed $k$-dependence of the formfactors is not important. The experimental $k$-distribution is shown in figure 3. Most of the experimental results presented here have been obtained with a collimator 1.7° × 2.6°, i.e. slightly narrower than that corresponding to figure 3.

When $\sqrt{-t} < 2.4$ fm$^{-1}$ the calculated formfactors may be parametrized as

$$S^-(t) = 1.31 \exp(-1.38 \text{ fm} \sqrt{-t}) - 0.31 \exp(-7.30 \text{ fm} \sqrt{-t})$$

(10)

$$S^+(t) = S^-(t) \exp(-0.13 \text{ fm}^2 t)$$

(11)
where we have normalized to 1 at \( t = 0 \, \text{fm}^{-2} \). The ratio \( F(t) \) expresses the emphasis which the \((d,2p)\) probe puts on the longitudinal channel. At typical values of the four momentum transfer \( F(t) \) can be of appreciable magnitude, for example \( F(-4 \, \text{fm}^{-2}) = 2.8 \). Thus the unpolarized cross section can be dominated by the longitudinal mechanism even when the interaction is predominantly transverse. The difference between the longitudinal and transverse formfactors \( S^+ \) and \( S^- \) arises from the interference between the the \( ^3S_1 \) and \( ^3D_1 \) states of the deuteron.

We have chosen to normalize the plane wave formfactor to 1 at \( t = 0 \, \text{fm}^{-1} \), even in the case where an explicit wave function is used in the calculation. This makes the comparison with other data, such as \((p,n)\) and \(^3\text{He},t\), easier.

The integral over the relative momentum \( k \) of the two protons in (9) is done once and for all, using a typical experimental distribution of \( k \) as represented by the calculated curve in figure 3. Doing so, we have neglected the \( |\mathbf{p}_{2p}| \)-dependence of the experimental cuts from the spectrograph: as the momentum of the \( 2p \) decreases, the cuts from the spectrograph become narrower. The cuts scale like \( |\mathbf{p}_{2p}| \) for each of the three dimensions of \( k \). In all, this calls for a geometric efficiency correction \( |\mathbf{p}_{2p}|^{-3} \). In the \( \Delta \) region this correction can be large, as an example, at deuteron bombarding energy \( T_{\text{lab}} = 2 \, \text{GeV} \) and energy transfer \( \omega = 300 \, \text{MeV} \) one obtains \( (|\mathbf{p}_{2p}|/|\mathbf{p}_d|)^{-3} = 1.4 \).

The correction comes on top of the efficiency correction within the 7% acceptance of each momentum setting of the spectrograph. The efficiency, within the 7% momentum acceptance of the spectrograph, was measured by stepping the central spectrograph momentum around the elastic peak value in the \( p(d,2p)n \) reaction keeping the scattering angle fixed.

Both corrections have been done in all shown spectra. By performing the integral over final state phase space with the cuts from spectrometer and collimator imposed on the integration region, averaging over initial momenta as emitted by the synchrotron, \( |\mathbf{p}_{2p}|^{-3} \)-correcting the result, and then comparing with the effective formfactor result, this has proven valid on the 2% level. The polarization response is formed as a ratio of cross sections and is insensitive to these procedures.

If we choose \( k = 0.12 \, \text{fm}^{-1} \), the plane wave formfactor \( S^\pm(k,t) \) is, apart from normalization, almost identical to the effective plane wave formfactor (9). In this sense the choice \( k = 0.12 \, \text{fm}^{-1} \), corresponding to an excitation energy of the di-proton of \( E_{2p} = 0.6 \, \text{MeV} \), represents the mean value of the relative momentum of the two ejectile protons for the experimental setup used in the results presented here. It is therefore the good choice for the asymptotic behavior of the \( 2p \) wavefunction in calculations where explicit wave functions are needed. In the calculations including absorption it has been checked that the absorption factor does not change the \( k \)-distribution, the absorption factor is effectively independent of \( k \) for the range of \( k \) of interest here.
2.3 Reference targets

The proton and deuteron targets serve as reference targets. The plane wave expression for the cross section in the "elastic" reactions is

\[
\frac{d\sigma}{dt} = 4 \frac{1}{3} \left( \frac{m_d}{m_n} \right)^2 \frac{1}{64 \pi F^2} \left\{ (|\beta|^2 + |\epsilon|^2 + |\gamma|^2) |S^- (t)|^2 + |\delta|^2 |S^+ (t)|^2 \right\}
\]

in terms of the nucleon-nucleon isospin amplitudes [4]. The phenomenological amplitudes [17] are plotted in figure 13.

In figure 4 the measured angular distributions on proton and deuteron targets are shown. The distribution predicted in the plane wave approximation is seen to reproduce very well the data all the way out to momentum transfer of 2.4 fm\(^{-1}\).

In the experiment \(\cos 2\varphi \approx 1\) gives a reasonable description. At very forward angles it is not a good approximation. But here \(T_{22}^M\) is small by symmetry around the beam axis, so the error we make by the assumption is not big, and certainly good enough to explain the physical content of the measured analyzing power. This leads to the expression

\[
P \approx \rho^{beam}_{20} \frac{1}{\sqrt{2}} \frac{2 (|\beta|^2 + |\gamma|^2) - (|\epsilon|^2 + |\delta|^2 F(t))}{|\beta|^2 + |\epsilon|^2 + |\gamma|^2 + |\delta|^2 F(t)}
\]

for the combination of analyzing powers we have determined experimentally. Since, phenomenologically, the amplitude \(\varepsilon\) is small, the measured quantity gives a quite good separation between the spin longitudinal \(\delta\) and the spin transverse \(\beta\). The simple picture is: \(P\) is large positive for transverse reaction mechanism, and large negative for spin longitudinal reaction.

The measured polarization response is also quite well described in the plane wave description at low momentum transfers. At larger momentum transfers the agreement is not so impressive. However, the Glauber calculation which includes scattering of the spectator in the spin-scalar channel gives a much improved description of the data, lending us confidence in our understanding of the probe (see section 4 for a discussion on the rescattering calculation).
Figure 4: a) The measured tensor polarization response on proton and deuteron targets. The dashed line is the plane wave prediction using the phenomenological nucleon-nucleon amplitudes at 800 MeV laboratory bombarding energy [17]. The full includes rescattering in the Glauber approach on the proton target.

b) The measured differential cross section on proton and deuteron target. The proton data have been normalized to the Glauber calculation at \( q = 0.7 \text{ fm}^{-1} \), corresponding to 0.85 times the PW normalization. Disregarding the zero degree point where the cross section is reduced on the deuteron because of Pauli blocking of the final state, the ratio between cross section on deuteron and proton targets is 0.68 ± 0.04.
3 Spin structure of the $\Delta$ excitation

The following section is mainly devoted to the discussion of the NN $\rightarrow$ N$\Delta$ reaction and the connection to the $p(d,2p)\Delta^+$ and $n(d,2p)\Delta^-$ reactions. But also the other elementary reaction $np \rightarrow pn$ of interest for the discussion of the spin isospin response of nuclei will be considered.

Only little is said about the excitation of the $\Delta$ in the nucleus. Effects of distortion have not yet been evaluated in this case. But, since the effects are considerable in the quasifree nuclear region, they should be expected to be of similar importance in the quasifree delta region.

At forward scattering and finite laboratory energy transfer $\omega$ the only non-vanishing analyzing power is

$$T_{20}^m \simeq T_{20}^B = \frac{1}{\sqrt{2}} \frac{|\beta|^2 + |\epsilon|^2 - 2 |\delta|^2 F(t)}{|\beta|^2 + |\epsilon|^2 + |\delta|^2 F(t)} = \frac{1}{\sqrt{2}} \frac{(T/L) - 2 F(t)}{(T/L) + F(t)} \tag{16}$$

where it is indicated that the response is a function of the ratio $(T/L)$ of transverse to longitudinal cross section in the underlying $(n,p)$ transition. $F(t)$ is the squared form factor ratio defined in equation (12).

In PWIA (ignoring Fermi motion in the deuteron target) the cross section for $\Delta$ production on the $p$ is expected to be proportional to the cross section on the $d$

$$d\sigma_{d(d,2p)\Delta^-} = 4 \, N \, d\sigma_{p(d,2p)\Delta^+} \tag{17}$$

where the factor $4 = 3 + 1$ represents the sum of the isospin factors for two possible final states with the deuteron target, and $N$ is an absorption factor which is taken from the ratio of "elastic" cross sections, figure 4. Experimentally we find $N = d\sigma_{d(d,2p)2n}/d\sigma_{p(d,2p)n} = 0.68 \pm 0.04$, except at $\theta_{lab} = 0^\circ$ where Pauli blocking in the triplet $2n$ final state causes some reduction of the cross section. In figures 6b, 7b, 8b, 9b, and 10b, the cross section on $d$ target has been scaled down by the factor $4 \times 0.68$. The remarkable agreement shows that the absorption factor is common for the low lying region and the $\Delta$ region.

Since, furthermore, as can be seen from figures 6a, 7a, 8a, 9a, and 10a, the polarization response is equal on the $p$ and $d$ targets, we feel comfortable in saying that the reaction is dominated by a simple one step process as indicated in figure 1: The exchange term on the $n$ target, and the contribution from projectile excitation of the $\Delta$ resonance cannot be large.

3.1 One pion exchange and Poor Mans Absorption model

In order to give some feeling of the origin of the strong transverse contributions to the scattering process, let us make a small excursion into one of the "absorption" models developed in the 60’es [19, 20, 21, 22].

The pure one pion exchange description of the NN $\rightarrow$ N$\Delta$ reaction is taken from the review paper of Dmitriev et al. [24]. The $\pi$NN and $\pi$N$\Delta$ vertices are

$$\pi\text{NN} : \quad ig_\pi \, \bar{u}(p_3,h_3) \gamma_5 u(p_1,h_1) \, \tau \cdot \pi \tag{18}$$
$$\pi\text{N}\Delta : \quad i \frac{f_\pi}{m_\pi} \bar{u}_\mu(p_4,h_4) u(p_2,h_2) \, q_\mu \, T \cdot \pi \tag{19}$$
where the convention for the $\gamma$-matrices is that used by Sakurai [25]. Details of the calculation of the diagrams are given in appendix C. In the Born approximation the scattering matrix $M$ is the product of the two vertex factors multiplied by the pion propagator and a factor $(2m_\pi)^{1/2}$ for each external fermion. This gives the normalization

$$\frac{d\sigma}{dt} = \frac{1}{64\pi F^2} |M_{fi}(s,t)|^2$$

(20)

$$F^2 = p_1p_2 - (m_1m_2)^2$$

(21)

for the transition from a specific initial to specific final state. The coupling constants used are $g_\pi = 13.61$ and $f_\pi^* = 2.0 m_\pi/(2m_N)$ for each external fermion. This gives the normalization for the transition from a specific initial to specific final state. The coupling constants used are $g_\pi = 13.61$ and $f_\pi^* = 2.0 m_\pi/(2m_N)$ for each external fermion.

In order to take into account the finite size of the nucleon and delta, a formfactor

$$\Gamma_\pi(t) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t}$$

(22)

is ascribed to each vertex. The same cut-off mass is used in both vertices, typically $\Lambda_\pi = 550 - 600$ MeV is needed to get a consistent description of the $n + p \rightarrow p + n$ and $p + p \rightarrow n + \Delta^{++}$ reactions. The cut-off mass is the crucial parameter in fitting the angular distributions to the experiment. The finite width of the $\Delta$ is inessential for the discussion of the spin response, the description of it from [24].

### 3.1.1 Poor mans absorption model

Focussing on the spin-isospin channel of the NN interaction, the direct $\pi$-exchange is taken as the fundamental mechanism. The $\pi$-pole at $t = m_\pi^2$ lies sufficiently close to the physical region, that the pole contribution may be expected to be the dominant term in the scattering process, at least for momentum transfers not bigger not bigger than a few $m_\pi$.

The short range repulsion of the strong interaction in the spin- and isospin-scalar channel, possibly combined with shadowing effects, prevents the colliding nucleons from probing the $\pi$-exchange at very short distances. Some authors discuss the possibility, that the origin of the short range repulsion is a symmetry property arising from the fact that the total 6-quark wavefunction must be symmetric under exchange of any two quarks [28, 29].

At high energies the impact parameter is a good measure of the closeness of the scattering process. The poor mans absorption model provides a recipe to change the amplitude at small impact parameters. The impact parameter spans a two dimensional space, thus leading to two dimensional modifications of the scattering amplitude. This contrasts the low energy result where the familiar minimal cut is symmetric in three dimensions, i.e. the $\pi$-exchange $+ \frac{1}{3} (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2)$ interaction.

A regularized helicity amplitude $B(s,t)$ without the propagator and without the kinematical singularities at the $s$-channel physical boundary is defined through

$$\langle h_3 h_4 | M(s,t) | h_1 h_2 \rangle = b_{h_1 h_2}(x) \frac{\langle h_3 h_4 | B(s,t) | h_1 h_2 \rangle}{m_\pi^2 - t}$$

(23)

where

$$b_{h_1 h_2}(x) = \left( \frac{1 - x}{2} \right)^{\frac{|h_1 - h_2|}{2}} \left( \frac{1 + x}{2} \right)^{\frac{|h_1 + h_2|}{2}}$$

(24)
contains the kinematic singularities in the cosine of the CM scattering angle \( x \) [30, 31]. The constraint that as \( x \to \pm 1 \), \( \mathcal{M} \propto b_{h_h f}(x) \) expresses the degree of helicity conservation associated with the conservation of the total angular momentum.

The regularized amplitude \( \mathcal{B}(s, t) \) is a polynomial in \( t \). The behavior of the amplitude at small impact parameters is determined by the behavior of \( \mathcal{B}(s, t)/(m^2 - t) \) at large \((-t)\). In order to exhibit this behavior the amplitude is expanded around the pole

\[
b_{h_h f}(x) \frac{\mathcal{B}(s, m^2_x) + \ldots}{m^2 - t}
\]

(25)

All terms, except the 0th order term, has large components at large four momentum transfer and are responsible for the contributions at small impact parameters. In the poor mans absorption model 1st and higher order terms are omitted under the assumption, that the strong short range repulsion in the elastic channel, whatever its origin, prevents the amplitude from being probed at small impact parameters.

### 3.1.2 On shell approximation

The procedure used to couple the spin of projectile nucleon to the spin of the spectator nucleon of the probe in the \((d, 2p)\) is described in appendix A.3. Since the PMA is calculated in the NN center of mass system, it involves Wigner rotations, to the probe Breit frame, of the spins of the probe nucleon. The approximation in the calculation of the angles is to regard the NN \( \to N\Delta \) reaction to have its trajectory on the energy shell. In order to avoid non-physical rotations it is then necessary to do the calculation at a virtual bombarding energy somewhat higher than the energy per nucleon in the \((d, 2p)\) reaction.

The rotation from the NN CM frame (where the elementary process is calculated) to the Breit frame (where the formfactor is multiplied) is almost balanced by the rotation from the Breit frame to the Madison frame. Thus, the rotation is important only because the \((d, 2p)\) formfactors are so different.

### 3.1.3 Comparison with experiments

As noted in [4] OπE-PMA applied to the elementary charge exchange reaction \( n + p \to p + n \) does well in describing the balance between spin longitudinal and spin transverse amplitudes even at bombarding energy as low as \( T_{\text{lab}} = 800 \text{MeV} \). Furthermore we find, that the \( t \)-dependence of these amplitudes as well as the forward differential cross section comes out in qualitative agreement with data when the cut-off mass \( \Lambda_x \approx 600 \text{MeV} \). In figure 13 the comparison of the \( T_{\text{lab}} = 800 \text{MeV} \) phenomenological amplitudes [17] with the poor mans absorption model is shown. In the model, only the longitudinal amplitude \( \delta \) and one transverse amplitude \( \beta \) are non zero. They agree qualitatively with the experimental amplitudes. And, indeed, the other phenomenological amplitudes are small at low momentum transfers. The smallness of the \( \epsilon \) amplitude is emphasized: it is zero with the poor mans cut, whereas, with the minimal cut model the amplitudes \( \epsilon, \beta \) and \( \delta \) coincide at momentum transfer \( q = 0 \text{fm}^{-1} \). The elastic charge exchange is further commented in section 4.2
In figures 6, 7, 8, 9, and 10, we compare the elementary $\Delta$ production data, obtained in the $(\bar{d}, 2p[1S_0])$ reaction, with the pure pion exchange model as given in [24]. Also, the poor mans absorption prescription applied to the one pion exchange is shown.

With a cut-off mass in the monopole vertex form factors of $\Lambda_\pi = 550\text{ MeV}$, equal at the $\pi NN$ and $\pi N\Delta$ vertices, the $\Omega\pi E$ model describes quite well the cross section and angular distribution. However, it fails completely in the description of the polarization response.

The $\Omega\pi E$-PMA model does quite well at large scattering angles, in both reproducing qualitatively the experimental polarization response and cross section. At forward scattering the model fails: it overestimates the cross section a factor $\sim 2.5$ and it has much larger spin longitudinal contribution than experimentally observed.

In figure 6 the $\theta_{\text{lab}} = 0^\circ$ data is compared with the modified $\pi + \rho$ exchange model with minimal cut. The parameters for the model are taken from [23]. In this reference the authors demonstrated that the model fails to reproduce the cross section. Here we demonstrate that it produces a zero crossing in the longitudinal amplitude at $-t \simeq m_\pi^2$, thus making the response purely transverse at laboratory energy transfer $\omega \sim 175\text{ MeV}$. This is clearly not seen experimentally.

In figure 11a the $\Omega\pi E$ and the $\Omega\pi E$-PMA models are compared with $p(p, n)\Delta^{++}$ data [27]. At incoming momentum $p_{\text{lab}} = 3.0\text{ GeV/c}$ the $\Omega\pi E$-PMA still fails at forward scattering, giving factor $\sim 2$ too much cross section. (Note, that the forward cross section is essentially independent of $\Lambda_\pi$.) At $p_{\text{lab}} = 11.75\text{ GeV/c}$ it does very well. At $-t$ larger than $25\text{ fm}^{-2}$ the slope of the measured cross section changes dramatically, indicating that higher order processes take over (two pion exchange?).

In figures 11 b-d and 12 b-d measured spin density matrix elements $\rho_{33} = \langle \frac{3}{2}, \rho | \frac{3}{2} \rangle$, $\rho_{31}$ and $\rho_{3-1}$ of the decaying $\Delta^{++}$ in the $pp \rightarrow n\Delta^{++}$ reaction are shown. The spin index is the helicity of the decaying $\Delta$ in the overall CM system. Also, the pure one-pion-exchange model and the poor mans absorption model used on the one-pion-exchange are shown. At small four momentum transfer the $\Omega\pi E$-PMA agrees astonishingly well with the data, when energies are higher than $3\text{ GeV}$. Especially it is emphasized, that the dip in $\rho_{33}$ is reproduced in both position and magnitude: The density-matrix element $\rho_{33}$ measures, at very forward angles, the ratio between spin transverse and spin longitudinal cross section, $T/(T + L) \approx \frac{3}{5} \rho_{33}$. The $pp \rightarrow n\Delta^{++}$ data therefore confirms the absence of a zero crossing in the spin longitudinal channel. At large $(-t)$, where other than S-wave in the quark wavefunctions of the nucleon and delta can contribute to the vertex form factors, there is indication of interference between different components of the intrinsic wave functions, showing up as oscillations around the PMA curve in figure 12.

Altogether this suggests that: When kinematics is elastic, and bombarding energy is intermediate ($\sim 1\text{ GeV}$) or higher, the spin-isospin channel of the $NN \rightarrow NN$ and the $NN \rightarrow N\Delta$ reactions are qualitatively described by the one pion exchange mechanism with a small impact parameter cut as given by the poor mans prescription. The cut-off mass $\Lambda_\pi$ needed is $\sim 600\text{ MeV}$. The goal is, of course, to understand what is the origin of the short range "screening" in the hope to be able to extend the description to the highly inelastic excitation of $\Delta$'s below $1\text{ GeV}$ bombarding energy per nucleon.
3.2 The nuclear response in the $\Delta$ region

In figure 5 the results at scattering angle $\theta_{\text{lab}} = 0^\circ$ and bombarding energy $T_{\text{lab}} = 2\text{ GeV}$ is shown. The shift of the $\Delta$ peak is $\sim 65\text{ MeV}$, and thus confirms the shift observed with the $(^{3}\text{He},t)$ probe [6].

On all three targets $p$, $d$ and $^{12}\text{C}$ the polarization response is well parametrized over the $\Delta$ region assuming constant ratio of transverse to longitudinal cross section in the underlying $(n,p)$ transition. The measured ratios are $(T/L)_p = 1.93 \pm 0.22$, $(T/L)_d = 1.71 \pm 0.09$, and $(T/L)_C = 2.99 \pm 0.23$.

For the elementary reactions, i.e. $p(d,2p)^\Delta$ and $d(d,2p)^n\Delta^-$, the spin transverse cross section dominates over the spin longitudinal. Contrary to the naive expectation [7, 8] we find, that on the nucleus, the transverse cross section is enhanced relative to the longitudinal [16].

As yet no calculation of the nuclear response taking properly into account effects of absorption on the spin observables in the $\Delta$ region has been done. However, the method used in the quasifree region described in section 4 should be applicable.

As far as I can judge at this moment, the effect of absorption will give rise to a relative enhancement of the spin transverse cross section over the spin longitudinal on the nuclear target. The apparent conflict between the spin response response in the elementary production of the $\Delta$ and the production in the nucleus is likely to be, at least in part, an effect of distortion.
Figure 5: a) Tensor polarization response (4) as measured on d and $^{12}$C targets at bombarding energy $T_{lab} = 2\,\text{GeV}$ and forward scattering. In the $\Delta$ region the calculated polarization response in $(d,2p)$, assuming constant ratio of transverse to longitudinal cross section in the underlying $(n,p)$ transition is shown for purely longitudinal and purely transverse transition, and the best fits for d and $^{12}$C. In the low $\omega$ region the elastic response from the $p$ target, $\nabla$, is shown to coincide with the Gamow-Teller transition $^{12}$C $\rightarrow$ $^{12}$N as expected in PWIA.
b) Cross section versus laboratory energy transfer for $(d,2p)$ on $p$, $d$ and $^{12}$C targets. The absolute scale is obtained by normalization to the $p(d,2p)n$ peak as in [1]. Also, the $p$ spectrum multiplied by $4\times0.68$ as suggested in (17) is shown. At $\omega = 13\,\text{MeV}$ the Gamow-Teller peak is seen. In the low lying region cross sections have been divided by 50.
Figure 6: a) Polarization response on $p$ and $d$ targets at $T_{\text{lab}} = 2\text{GeV}$ and $\theta_{\text{lab}} = 0^\circ$ where $\langle \cos 2\varphi \rangle = -0.28$. The curves are direct $\pi$-exchange (full), $\pi$-exchange with PMA short range cut (dash), $\pi$-exchange with $\frac{3}{4}(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2)$ cut (short dash), and modified $\pi + \rho$ exchange with minimal short range cut [23] (dots).

b) Cross section on $p$ (full step) and $d$ (dash step) targets. The $d$ cross section has been divided by $4 \times 0.68$. The models are the direct $\pi$-exchange (full curve) and the $\pi$-exchange with PMA cut (dash), both with cutoff mass $\Lambda_{\pi} = 550\text{MeV}$ in the monopole vertex form factors.
Figure 7: a) Polarization response on p and d targets at $T_{\text{lab}} = 2$ GeV and $\theta_{\text{lab}} = 2.1^\circ$ where $\langle \cos 2\varphi \rangle = 0.86$. The curves are direct $\pi$-exchange (full), $\pi$-exchange with PMA short range cut (dash).

b) Cross section on p (full step) and d (dash step) targets. The d cross section has been divided by $4 \times 0.68$. The models are the direct $\pi$-exchange (full) and the $\pi$-exchange with PMA cut (dash), both with cutoff mass $\Lambda_\pi = 550$ MeV in the monopole vertex form factors.
Figure 8: a) Polarization response on p and d targets at $T_{\text{lab}} = 2\,\text{GeV}$ and $\theta_{\text{lab}} = 4.3^\circ$ where $\langle \cos 2\varphi \rangle = 0.96$. The curves are direct $\pi$-exchange (full), $\pi$-exchange with PMA short range cut (dash).

b) Cross section on p (full step) and d (dash step) targets. The d cross section has been divided by $4 \times 0.68$. The models are the direct $\pi$-exchange (full) and the $\pi$-exchange with PMA cut (dash), both with cutoff mass $\Lambda_r = 550\,\text{MeV}$ in the monopole vertex form factors.
Figure 9: a) Polarization response on $p$ and $d$ targets at $T_{\text{lab}} = 2\,\text{GeV}$ and $\theta_{\text{lab}} = 5.7^\circ$ where $(\cos 2\phi) = 0.98$. The curves are direct $\pi$-exchange (full), $\pi$-exchange with PMA short range cut (dash).

b) Cross section on $p$ (full step) and $d$ (dash step) targets. The $d$ cross section has been divided by $4 \times 0.68$. The models are the direct $\pi$-exchange (full) and the $\pi$-exchange with PMA cut (dash), both with cutoff mass $\Lambda_\pi = 550\,\text{MeV}$ in the monopole vertex form factors.
Figure 10: a) Polarization response on p and d targets at $T_{\text{lab}} = 2 \text{ GeV}$ and $\theta_{\text{lab}} = 7.2^\circ$ where $\langle \cos 2\varphi \rangle = 0.99$. The curves are direct $\pi$-exchange (full), $\pi$-exchange with PMA short range cut (dash).

b) Cross section on p (full step) and d (dash step) targets. The d cross section has been divided by $4 \times 0.68$. The models are the direct $\pi$-exchange (full) and the $\pi$-exchange with PMA cut (dash), both with cutoff mass $\Lambda_{\pi} = 550 \text{ MeV}$ in the monopole vertex form factors.
Figure 11: a) $p(p, n)\Delta^{++}$ cross section data at $p_{\text{lab}} = 3.0$ GeV/c [26], is compared with the OπE PMA model (full line), and with the pure OπE model (dash). In both cases the cutoff mass was chosen to $\Lambda_r = 600$ MeV.

b) c) d) The measured density matrix elements of the decaying $\Delta$ in the center of mass helicity representation are compared with the OπE PMA model (full line), and the pure OπE model (dash).
Figure 12: a) $p(p, n)\Delta^{++}$ cross section data at $p_{\text{lab}} = 11.75\text{GeV}/c$ [26] is compared with the OπE PMA model (full line), and the pure OπE model (dash). In both cases the cutoff mass was chosen to $\Lambda_\pi = 550\text{MeV}$.

b) c) d) The measured density matrix elements of the decaying $\Delta$ in the center of mass helicity representation are compared with the OπE PMA model (full line), and the pure OπE model (dash).
4 The quasifree region

Since Alberico, Ericson and Molinari pointed out the contrast between the effect of correlations in the spin longitudinal versus the spin transverse response calculated for nuclear matter within the RPA scheme, much interest has been devoted to this subject, experimentally as well as theoretically [32, 33],

The experimental observation of a large downwards shift of the quasifree bump with the \((\text{He}, t)\) reaction at large momentum transfers supported this hypothesis, provided a dominance of spin longitudinal reaction mechanism was assumed [34, 35].

The prediction of an enhancement of the spin longitudinal over the spin transverse response is contradicted by the investigation of the spin isospin response with the \((p, p')\) experiment [35, 36], and with the charge exchange reaction \((d, 2p)\). The online result of the recent \((p, n)\) seem to confirm the (apparent) conflict [11].

4.1 Distortion prescription when omitting spin dependence

In the following a simple, but rather powerful, distortion prescription shall be outlined. The aim of the calculation is to enable us to “subtract” the effect of distortion in order to be able to say if there remains measurable effects on cross section and spin-observables from a medium polarization.

In the eikonal limit of Glauber theory the \((d, 2p)\) reaction on nuclei is calculated. The force of the calculation is that it takes into account the absorption of the projectile nucleons individually. This is found to have substantial effects on the tensor spin observables in the reaction with polarized beam.

The calculation does not yet allow for inelastic rescattering of either of the nucleons in the probe. Exactly this limitation of the discussion may be the reason for the failure of the description at large energy transfers.

This limitation is shared with DWBA calculations of the \((p, n)\) reaction on nuclei in the quasifree region. The calculations (therefore?) have in common that they underestimate the cross section in charge exchange reactions at large energy transfers.

In the present section the general ideas are introduced, in 4.2 the spin dependence of the problem is included, and, finally, in 4.3 the eikonal absorption limit is described.

In the case of \((N, N)\) scattering on a nucleus the product of the incoming and outgoing distorted waves is written

\[
\chi_f^I(s_1) \chi_i(s_1) = e^{i \mathbf{q}_{\text{ex}} \cdot \mathbf{s}_1} D_1(s_1)
\]

\[e^{i \mathbf{q}_{\text{ex}} \cdot \mathbf{s}_1} = e^{i (\mathbf{k}_i - \mathbf{k}_f) \cdot \mathbf{s}_1}\] is the product of incoming and outgoing plane waves and the distortion function \(D_1\) describes the deviation from plane wave approximation.

In the case of \((d, 2p)\) the product of the incoming and outgoing waves is written

\[
\chi_f^I(s_1, s_2) \chi_i(s_1, s_2) = e^{i \mathbf{q}_{\text{ex}} \cdot (\mathbf{s}_1 + \mathbf{s}_2)/2} D_1(s_1) D_2(s_2) \psi_{\text{2p}}^I(s_2 - s_1) \psi_d(s_2 - s_1)
\]

where the intrinsic wavefunctions in entrance and exit channels are introduced. The distortion is allowed to act on the projectile particles separately through the distortion functions \(D_1\) and
So far the discussion is quite general, it only assumes that the effect of distortion on the projectile particles can be factorized. This approximation has previously been discussed in [9].

Even though the momentum representation will be preferred for the final expression, it is instructive to start out in configuration space. Here the scattering matrix writes

\[ T = \int d^3s_1 d^3s_2 d^3r \; e^{i \mathbf{q}_{\text{ex}} \cdot (\mathbf{s}_1 + \mathbf{s}_2)/2} \; D_1(s_1) D_2(s_2) \]

\[ \psi_{2p}^\dagger (\mathbf{s}_2 - \mathbf{s}_1) \phi_p^\dagger (\mathbf{r}) \; V(\mathbf{s}_1 - \mathbf{r}) \; \phi_h(\mathbf{r}) \; \psi_d(\mathbf{s}_2 - \mathbf{s}_1) \]

\( p \) denotes the particle state, \( h \) the hole state of the nucleus, and \( V \) is the nucleon-nucleon charge-exchange interaction.

In order to get the formulation in momentum representation we make the substitutions

\[ V(\mathbf{s}_1 - \mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} \; t(\mathbf{q}) \; e^{-i \mathbf{q} \cdot (\mathbf{s}_1 - \mathbf{r})} \]

\[ s_2 = s_1 + s \]

and obtain

\[ T = \int \frac{d^3q}{(2\pi)^3} \int d^3s_1 \; e^{i (\mathbf{q}_{\text{ex}} - \mathbf{q}) \cdot s_1} \; D_1(s_1) \]

\[ \int d^3s \; \psi_{2p}^\dagger (s) \; e^{i \mathbf{q}_{\text{ex}} \cdot s/2} \; D_2(s_1 + s) \; \psi_d(s) \]

\[ t(\mathbf{q}) \int d^3r \; \phi_p^\dagger (\mathbf{r}) \; e^{i \mathbf{q} \cdot \mathbf{r}} \; \phi_h(\mathbf{r}) \]

This will be taken as the starting point for the present distortion calculation.

The second line of (30) may be identified as an effective formfactor for the probe

\[ S(\mathbf{q}_{\text{ex}}, s_1) = \langle 2p \mid e^{i \mathbf{q}_{\text{ex}} \cdot s/2} D_2(s_1 + s) \mid d \rangle \]

\[ = \int d^3s \; \psi_{2p}^\dagger (s) \; e^{i \mathbf{q}_{\text{ex}} \cdot s/2} \; D_2(s_1 + s) \; \psi_d(s) \]

It contains all information about the influence of the spectator particle including its absorption when it passes through the target nucleus. When the impact parameter is much larger than the size of the target plus the size of the projectile, the distortion function \( D_2 \) is close to unity in the important part of the integration region: the effective formfactor becomes just the plane wave \((d, 2p)\) formfactor.

The integral in the third line of (30) is the target formfactor

\[ \langle p h^{-1} \mid \sum_{j=1}^{A} e^{i \mathbf{q} \cdot \mathbf{r}_j} \mid 0 \rangle = \int d^3r \; \phi_p^\dagger (r) \; e^{i \mathbf{q} \cdot \mathbf{r}} \; \phi_h(r) \]

which will be considered in the section on the nuclear response function.

The Fourier transform into momentum representation of the product of the distortion function and the effective formfactor

\[ f(\mathbf{q}_{\text{ex}}, \delta) = \int d^3s_1 \; e^{i \delta \cdot s_1} \; D_1(s_1) \; S(\mathbf{q}_{\text{ex}}, s_1) \]

25
evaluated at the deviation $\delta = q_{\text{ex}} - q$ from the external momentum transfer appears in the transition matrix element. $f$ is referred to as the probe weight function.

In the formalism just established the scattering matrix is an integral with weight $f$ of the product of the driving force and the formfactor for the transition in the target nucleus

$$T = \int \frac{d^6 \delta}{(2\pi)^3} f(q_{\text{ex}}, \delta) t(q_{\text{ex}} - \delta) \left( p \hbar^{-1} | \sum_{j=1}^{A} e^{i(q_{\text{ex}} - \delta) \cdot r_j} \right)$$

(34)

At this level the spin and isospin dependence is not included.

4.2 Including spin

In the following the spin dependence of the driving force and the dependence of the deuteron orientation is included in the absorption prescription. However, the spin dependence (the spin orbit potential) of the absorption functions $D_1$ and $D_2$ is not included since it has only small effect on the tensor spin observables [4]. Further it is not included in the eikonal absorption limit which will be made below.

4.2.1 Driving force

Figure 13: Phenomenological $n+p \rightarrow p+n$ charge exchange amplitudes [17] compared with the poor mans absorption model applied to one pion exchange. The cut-off mass $\Lambda_r = 600$ MeV was used in the monopole vertex form factor. Only two of the five amplitudes allowed by time reversal invariance and parity conservation of the strong interaction come out of the model. We emphasize the smallness of the $\varepsilon$ amplitude at forward scattering which in the one pion exchange model with poor mans cut is zero. The amplitudes have been divided by $(2m_n)^2$ relative to the normalization in (39).
In most calculations, as in the present, the driving force (the external field) is represented by phenomenological nucleon–nucleon amplitudes, in the case of quasi elastic \((d,2p)\) by the \(n + p \rightarrow p + n\) charge exchange amplitudes. The advantage of this is that they are measured, and therefore it is not necessary to rely on models for this part of the calculation. However, the amplitudes are only measured on the energy shell, and are for given masses of the particles only functions of the total invariant energy \(\sqrt{s}\) and the four-momentum transfer \(\sqrt{-t}\). The approximation to use the on-shell amplitudes in place of the off-shell amplitudes is used. \(^1\)

The spin dependence of the driving force is described by resolving the interaction into its spherical tensor components. The rank \(\sigma = 0, 1\) and the component \(\mu = -\sigma, ..., \sigma\) of the interaction is specified at both “vertices”

\[
\langle pn | t^0(q) + t^1(q) \tau^1 \cdot \tau^2 | np \rangle = 2 t^1(q) = 2 \sum_{\sigma_1 \mu_1 \sigma_2 \mu_2} t_{\sigma_1 \mu_1 \sigma_2 \mu_2}(q) \sigma_{\sigma_1 \mu_1}^\dagger \sigma_{\sigma_2 \mu_2}^\dagger \tag{35}
\]

The spin dependence of the interaction is in the operators \(\sigma_{\sigma \mu}\) acting in the two vertices of the process. Since we are dealing with \(np \rightarrow pn\) scattering the operators must connect spin \(\frac{1}{2}\) with spin \(\frac{1}{2}\), and only rank \(\sigma = 0, 1\) can contribute. The rotational invariance is explicit since (35) is a scalar product at each vertex. The superscript 1 indicating that only the isospin (transfer) channel of the nucleon-nucleon interaction contributes, will be suppressed in the following. Note that, as opposed to the more general expression (5), the spin transition operator is explicit in the target “vertex” also.

The five independent amplitudes for each isospin channel are sometimes given in cartesian CM components \([37, 17]\), in the charge exchange channel they are denoted by Greek letters

\[
t_{CM}^{\sigma}(q) = \alpha + \beta \sigma_Y^1 \sigma_Y^2 - i \gamma (\sigma_Y^1 + \sigma_Y^2) + \delta \sigma_X^1 \sigma_X^2 + \varepsilon \sigma_Z^1 \sigma_Z^2 \tag{36}
\]

where the co-ordinate system has \(x, y\) and \(z\) axes along \(-q = k_f - k_i, n = k_i \times k_f, \) and \(-q \times n\). The initial and final state momentum vectors \(k_i\) and \(k_f\) are taken in the nucleon-nucleon CM-system. The phenomenological \(np \rightarrow pn\) amplitudes at 800 MeV bombarding energy \([17]\) are plotted in figure 13. The normalization is such as to give

\[
\frac{d\sigma}{dt} = \frac{1}{64\pi F^2} 4 \left( |\alpha|^2 + |\beta|^2 + 2 |\gamma|^2 + |\delta|^2 + |\varepsilon|^2 \right) \tag{37}
\]

\[
F(s) = ((k_1 k_2)^2 - m_n^4)^{\frac{1}{2}} = m_2 |p_{lab}| = \sqrt{s} |k| \tag{38}
\]

This “invariant” normalization lead to the non-relativistic expression

\[
t = - \left( \frac{f_\pi}{m_\pi} \right)^2 \left( 2 m_n \right)^2 \frac{(\sigma^1 \cdot q)(\sigma^2 \cdot q)}{m_n^2 - t} \tau^1 \cdot \tau^2 \tag{39}
\]

for the one pion exchange diagram. The poor mans absorption model, discussed in section 3.1.2 for the \(\Delta\)-region, gives when applied to the \(O\pi E\)

\[
t = \frac{1}{2} \left( \frac{f_\pi}{m_\pi} \right)^2 \left( 2 m_n \right)^2 \left\{ \frac{m_n^2 - q^2}{m_n^2 + q^2} (\sigma^1 \cdot q)(\sigma^2 \cdot q) + (\sigma^1 \cdot \hat{n})(\sigma^2 \cdot \hat{n}) \right\} \tau^1 \cdot \tau^2 \tag{40}
\]

\(^1\)A discussion on the validity of this approximation may be found in the paper by Kerman, McManus and Thaler \([37]\).
With a formfactor
\[ \Gamma_\pi(t) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t} \]
\( \Lambda_\pi = 600 \text{ MeV} = 3.04 \text{ fm}^{-1} \) (41)
multiplied at each vertex this gives the qualitative behaviour of the amplitudes. The comparison with the phenomenological amplitudes is given in figure 13. The magnitude of the amplitudes \( \beta \) and \( \delta \) are qualitatively reproduced. However, the phase of the phenomenologically determined \( \beta \) precess somewhat at large momentum transfers, pushing some of the strength into the imaginary part.

If the two spin transverse amplitudes \( \beta \) and \( \epsilon \) were equal the interaction would be local (apart from the small spin-orbit interaction \( \gamma \)). This is a decent approximation at lab. bombarding energies below 300 MeV. At 800 MeV it is not a good approximation since here the \( \epsilon \) amplitude is very small. The difference between \( \beta \) and \( \epsilon \) gives rise to a non-locality of the nucleon-nucleon charge exchange interaction, as does the spin-orbit amplitude \( \gamma \) at larger momentum transfers.

The transformation into spherical components of the amplitudes (36) is
\[ t_{00}^{CM} = \alpha \]
\[ t_{10}^{CM} = \epsilon \]
\[ t_{11}^{CM} = t_{1-1}^{CM} = -\frac{1}{2}(\delta + \beta) \]
\[ t_{11}^{CM} = t_{1-1}^{CM} = \frac{1}{2}(\delta - \beta) \]
\[ t_{10}^{CM} = t_{-10}^{CM} = t_{01}^{CM} = t_{-01}^{CM} = -\frac{1}{\sqrt{2}} \gamma \]

The transformation from the nucleon-nucleon CM system with \( y \)-axis along \( \mathbf{k}_i \times \mathbf{k}_f \) to the lab. frame with \( Y \)-axis along \( \mathbf{p}_1 \times \mathbf{p}_3 \) is to a good approximation a rotation \( \psi_q = \varphi_q - \varphi_{\text{exc}} = \varphi_q - \pi \) around the common \( Z \)-axis
\[ t_{\sigma_1 \mu_1 \sigma_2 \mu_2}(q) \simeq e^{i\psi_q (\mu_1 + \mu_2)} t_{\sigma_1 \mu_1 \sigma_2 \mu_2}^{CM}(q) \] (43)
The \( Z \) axis is along the velocity \( \beta \) of the Breit frame as seen in the laboratory frame. This is identical to the velocity of the CM of the beam-ejectile system. The first three of the amplitudes in (42) are not affected by this transformation, whereas the latter two have their phase modified.

The approximation assumes that the momentum transfer parallel to \( Z \) is much smaller than the perpendicular momentum transfer, which is satisfied in the quasi elastic region at bombarding energies as high as 800 MeV per nucleon. A discussion of a more careful transformation of the amplitudes in the helicity representation is given in appendix A.3. It is important when the process is highly inelastic, as is the case for the \( \Delta \) excitation.

4.2.2 Probe wave functions

The deuteron wavefunction consists of a \( ^3S_1 \) and a \( ^3D_1 \) wave part. The radial wave functions wavefunctions \( U (= U_0) \) and \( W (= U_2) \) are taken from the Paris parametrization [38]
\[ \psi_d(s) = \frac{U(s)}{s} Y_{00}(s) \left( \left( \frac{1}{2} \right) 1M \right) + \frac{W(s)}{s} \left( Y_2(s) \left( \left( \frac{1}{2} \right) 1M \right) \right)_{(21)1M} \]
Figure 14: Di-proton wavefunction $u_k$ corresponding to $k = 0.12 \text{fm}^{-1}$, dashed curve, units on the right axis. Paris parametrization of the deuteron $^3S_1$ and $^3D_1$ wavefunctions, units on the left axis.

The $^1S_0$ state of the $2p$-system is described by the wavefunction

$$\psi_{2p}(s) = \frac{u_k(s)}{s} Y_{00}(s) |(1\frac{1}{2})00\rangle = \frac{u_k(s)}{s} Y_{00}(s) |00\rangle$$

where the radial wavefunction $u_k$ is a solution of the Schrödinger equation

$$\left( -\frac{1}{2\mu} \frac{d^2}{ds^2} + V_{\text{Reid}}(s) + \frac{e^2}{s} \right) u_k(s) = \frac{k^2}{2\mu} u_k(s)$$

$V_{\text{Reid}}$ is the Reid soft-core parametrization of the isospin triplet, spin singlet nucleon-nucleon interaction [39]. $u_k(s)$ may be normalized to

$$u_k(s) \sim \frac{\sqrt{4\pi}}{k} \sin(k s - \eta \log(2k s) + \sigma_0 + \delta) \quad \text{as} \ s \to \infty$$

corresponding to the plane wave box normalization $\langle k' | k \rangle = (2\pi)^3 \delta(3)(k' - k)$. ($\mu = \text{reduced mass}, \eta = (\mu e^2)/k, \sigma_0 = \text{Coulomb phase shift}, \delta = \text{nuclear phase shift}, [40].$) In the calculations the value $k = 0.12 \text{fm}^{-1}$ has been used for the internal momentum of the two protons of the ejectile. This value is the average value in the sense that the plane wave formfactor practically coincides with the effective plane wave formfactor (section 2.2). The numerical solution to the Schrödinger equation is shown in figure 14 where also the deuteron wave functions $U$ and $W$ are shown.
4.2.3 Scattering matrix including spin dependence

The effective $d \rightarrow 2p$ formfactor including the spin dependence and the triplet D state of the deuteron is then given by

$$\langle \mu_1 | S(q_{ex}, s_1) | M \rangle = \langle 2p[^1S_0] | \sigma_{\mu_1}^t e^{iq_{ex}\cdot s/2} D_2(s_1 + s) | dM \rangle$$

$$= \sum_{l=0,2} \langle l \beta | \mu_1 | 1M \rangle \int d^4s \frac{u(s)\dagger}{s} Y_00(s) e^{iq_{ex}\cdot s/2} D_2(s_1 + s) \frac{U_l(s)}{s} Y_{l\beta}(s)$$

where it was used that $\langle 00 | \sigma_{\mu_1}^t | 1m_s \rangle = \langle 00 \sigma_1 \mu_1 | 1m_s \rangle$ selects the spin transfer channel of the driving force at the probe "vertex", i.e. $\sigma_1 = 1$.

The Fourier transform of the product of the formfactor and the distortion function for the reactive particle is the probe weight function

$$\langle \mu_1 | f(q_{ex}, \delta) | M \rangle = \int d^3s_1 e^{iq_{ex}\cdot s_1} D_1(s_1) \langle \mu_1 | S(q_{ex}, s_1) | M \rangle$$

The expression for the scattering matrix becomes

$$T =$$

$$i \sum_{\mu_1 \sigma_2 \mu_2} \int \frac{d^2\delta}{(2\pi)^3} \langle \mu_1 | f(q_{ex}, \delta) | M \rangle t_{\mu_1 \sigma_2} (p, q_{ex} - \delta) \langle p h^{-1} | j_{\mu_1}^t \sigma_2 \mu_2 (q_{ex} - \delta) | 0 \rangle$$

where the spin-isospin current $j_{\tau \nu} \sigma_\mu$ is defined in (71).

4.3 Eikonal absorption limit of Glauber theory

In the eikonal limit the distortion functions are functions of the impact parameter only [9]

$$D_1(b_1) = \exp(-\bar{\rho}(b_1))$$

$$D_2(b_2) = \exp(-\bar{\rho}(b_2))$$

The factor $(A - 1)/A$ takes into account that the particle undergoing charge exchange is rescattered once less than others [9]. The effect of symmetrization of the $2p$ wavefunction on the absorption is ignored.

The nuclear thickness function $\bar{\rho}(b) = \int \rho((b^2 + z^2)^{1/2}) dz$ is taken from electron scattering data [41]. The parametrization used in the present calculation on the $^{12}$C target is

$$\rho_C(r) = \rho_0 (1 + a(r/r_0)^2) e^{-(r/r_0)^2}$$

$$\rho_0 = \frac{2A}{(\sqrt{\pi}r_0)^3(3a + 2)}$$
with parameters \( r_0 = 1.649 \text{ fm} \), \( A = 12 \), and \( a = 1.247 \). For the doubly closed shell nucleus \(^{40}\text{Ca}\)

\[
\rho_{\text{Ca}}(r) = \frac{\rho_0}{1 + e^{(r - R_0)/a}}
\]

(54)

with parameters \( \rho_0 = 0.176 \text{ fm}^{-3} \), \( R_0 = 3.510 \text{ fm} \) and \( a = 0.5630 \text{ fm} \) was used.

The distortion parameter \( \tilde{\gamma} \) is essentially taken from the optical theorem

\[
\tilde{\gamma} = -\frac{i}{2} \frac{4\pi}{p_{\text{lab}}} f_{\text{lab}}(0) = -\frac{i}{2} \frac{4\pi}{p_{\text{CM}}} f_{\text{CM}}(0)
\]

(55)

except that through the present prescription, the real part of the optical potential is also encountered. At \( T_{\text{lab}} = 800 \text{ MeV} \) per nucleon the value \( \tilde{\gamma} = (2.09 + i 0.29) \text{ fm}^2 \) is used (taken from [17]). In the Bohr-Peierls-Plazek limit the absorption parameter is \( 2\tilde{\gamma} = \sigma^\text{tot} = 4\pi/k \text{ Im } f(0) \).

The value \( \tilde{\gamma} = 2.09 \text{ fm}^2 \) corresponds to \( \sigma^\text{tot} = 41.8 \text{ mb} \). At bombarding energies larger than 600 MeV the real part of \( \tilde{\gamma} \) is found to be almost independent the bombarding energy. The imaginary part, which is of little significance to the numeric results in the present calculation, has some energy dependence. A possible reduction of the distortion parameter \( \tilde{\gamma} \) from Pauli blocking effects has been discussed in [42] and is found to be small at energies as high as of interest here.

Since, in the eikonal absorption limit, the distortion functions \( D_1 \) and \( D_2 \) are functions of the impact parameter of the probe particles the formulation essentially becomes two dimensional: The absorption does not change the component of the momentum transfer along the relative motion of the probe Breit frame with respect to the laboratory frame. In the following the trivial \( \delta \)-function in this direction has been suppressed, thus redefining the weight function \( f(q_{\text{ex}}, \delta) \)

\[
\langle \mu_1 | S(q_{\text{ex}}, b_1) | M \rangle = \sum_{l,m_1} \langle l\mu_1 | 1M \rangle \int d^2s \frac{u(s)^\dagger}{s} Y_{ll}^*(s) e^{i q_{\text{ex}} \cdot s/2} D_2(|b_1 + s_\perp|) \frac{U_l(s)}{s} Y_{lm_1}(s)
\]

\[
\langle \mu_1 | f(q_{\text{ex}}, \delta) | M \rangle = \int db_1 e^{i \delta \cdot b_1} D_1(b_1) \langle \mu_1 | S(q_{\text{ex}}, b_1) | M \rangle
\]

(57)

The function that is actually calculated numerically is

\[
\langle \mu_1 | \tilde{f}(q_{\text{ex}}, \delta) | M \rangle = \int db_1 e^{i \delta \cdot b_1} D_1(b_1) \langle \mu_1 | S(q_{\text{ex}}, b_1) - S(q_{\text{ex}}, \infty) | M \rangle
\]

(58)

which converges since the integrand vanishes exponentially at large \( b_1 \). Then \( f \) is given by

\[
\langle \mu_1 | f(q_{\text{ex}}, \delta) | M \rangle = (2\pi)^2 \delta^{(2)}(\delta) \langle \mu_1 | S(q_{\text{ex}}, \infty) | M \rangle + \langle \mu_1 | \tilde{f}(q_{\text{ex}}, \delta) | M \rangle
\]

(59)

i.e. a sum of a plane wave contribution and an interfering rescattering term which is smeared out around the external momentum transfer.
The range of $\tilde{f}(q_{ex}, \delta)$ regarded as a function of the distorting momentum transfer $\delta$ is of order $2\pi/(R_{target} + R_{projectile})$. This range is a measure of how much the momentum transfer in the charge exchange reaction is allowed to deviate from the plane wave momentum transfer $q_{ex}$. In figure 15, a contour plot of $\langle 0 | \tilde{f}(q_{ex}, \delta) | 0 \rangle$ for $q_{ex} = 1.30\text{fm}^{-1}$ is shown. In most of the rescattering phase space $\tilde{f}$ is negative indicating destructive interference between the plane wave and rescattering contributions. The weight function is considered in more detail in section 4.3.1.

Figure 15: Contour plot of the real part of $\tilde{f}(q_{ex}, \delta)$ on the $^{12}\text{C}$ target. The plot corresponds to the spin transverse part of $\tilde{f}$, i.e. the $\langle 0 | \tilde{f}(q_{ex}, \delta) | 0 \rangle$ component. The value of the external momentum transfer is $q_{ex} = 1.3\text{ fm}^{-1}$. The asymmetric distortion function is unique for a composite projectile. In the case of a point like probe the distortion function is symmetric around $(0,0)$.

With the above modifications the scattering matrix becomes

$$
\mathcal{M} = \sqrt{2} \times \left( \frac{m_1 m_2}{m_1 m_2} \right) \times

i \sum_{\mu_1 \sigma_2 \mu_2} \int \frac{d^2 \delta}{(2\pi)^2} \langle \mu_1 | f(q_{ex}, \delta) | M \rangle \ t_{1\mu_1 \sigma_2 \mu_2}(q_{ex} - \delta) \langle p h^{-1} | j_{1-1}^{\mu_1} \sigma_2 \mu_2(q_{ex} - \delta) | \hat{0} \rangle
$$

(60)
Here the, till this moment omitted, isospin matrix element $\langle p \mid \tau_{-1} \mid n \rangle = \sqrt{2}$ for the transition of the probe neutron into a proton is explicitly included. Further, a flux factor to bring us to the "invariant" normalization is multiplied.

4.3.1 Physical interpretation of the probe weight function

Figure 16: Eikonal limit of the distortion functions for the spectator (thin) and for the reactor (thick) in momentum space for the $^{12}$C target at 800 MeV bombarding energy per nucleon. The dashed line is the imaginary part.

If the distortion functions, $D_1$ and $D_2$, in configuration space are replaced by their Fourier transforms we may rewrite the probe weight function (57) as

$$\langle \mu_1 \mid f(q_{ex}, \delta) \mid M \rangle = (2\pi)^2 \delta^{(2)}(\delta) \langle \mu_1 \mid S(q_{ex}) \mid M \rangle \tag{61}$$

$$+ \widehat{D}_{1-1}(\delta) \langle \mu_1 \mid S(q_{ex}) \mid M \rangle$$

$$+ \widehat{D}_{2-1}(\delta) \langle \mu_1 \mid S(q_{ex} - 2\delta) \mid M \rangle$$

$$+ \int \frac{d^2\delta_2}{(2\pi)^2} \widehat{D}_{1-1}(|\delta - \delta_2|) \widehat{D}_{2-1}(\delta_2) \langle \mu_1 \mid S(q_{ex} - 2\delta_2) \mid M \rangle$$

The $\widehat{D}$ denotes the Fourier transform of $(D-1)$. The distortion functions in momentum space are plotted in figure 16 for the $^{12}$C target. They are of range inversely proportional to the nuclear radius. At 800 MeV bombarding energy the imaginary part is not of significance for the numerical results.
The plane wave \((d,2p)\) formfactor, a function of one variable only, is

\[
\langle \mu_1 | S(q) | M \rangle = \langle 2p[1S_0] | \sigma_{1\mu_1}^+ e^{i q \cdot s/2} | d1M \rangle
\]

\[
= \delta(\mu_1, M) \int_0^\infty u_k(s) j_0(\frac{q s}{2}) U(s) ds
\]

\[
- \langle 2(M-\mu_1) 1\mu_1 | 1M \rangle \sqrt{4\pi} Y_{2M-\mu_1}(q) \int_0^\infty u_k(s) j_2(\frac{q s}{2}) W(s) ds
\]

The expression (61) has a very simple interpretation. First line is the plane wave contribution. Second line represents rescattering on the reactor nucleon, a term which is symmetric around \(\delta = (0,0)\). These two contributions are common for a pointlike and a composite probe. The third and fourth line are specific for a composite probe. Third line is a term where the spectator is rescattered. The fourth line represents rescattering of both projectile nucleons. The PW probe form factor is in all cases evaluated at the difference between the momentum transfer on the reactor and the spectator.

Even though it would not be a good approximation, let us, for illustration, consider the extreme limit where the PW formfactor falls of much faster with \(q\) than other functions involved in the expression (60) for the scattering matrix. The probe weight function may then be replaced by

\[
(2\pi)^2 \delta(\delta)(\mu_1 | S(q_{ex}) | M)
\]

\[
+ D_{1-1}(\delta) \langle \mu_1 | S(q_{ex}) | M \rangle
\]

\[
+ D_{2-1}(\frac{q_{ex}}{2}) a \langle \mu_1 | S(0) | M \rangle (2\pi)^2 \delta(\delta) q_{ex} - 2\delta
\]

\[
+ D_{1-1}(\frac{q_{ex}}{2}) D_{2-1}(\frac{q_{ex}}{2}) a \langle \mu_1 | S(0) | M \rangle
\]

with \(a \approx 0.11 \text{ fm}^{-2}\) evaluated as the integral under the effective formfactor (10).

Now, take a look at the contour plot of \(f\) in figure 15, recalling that the external momentum transfer is to a good approximation along the negative X-axis. The term involving rescattering of both nucleons, line 4, gives rise to the constructively interfering term around \(\delta_x = -1.1 \text{ fm}^{-1}\). The quite sharply peaked destructively interfering term around \(\delta_x = -0.5 \text{ fm}^{-1}\) corresponds to rescattering of the spectator nucleon only, in the schematic expression above it is approximated by a Dirac delta function. The rescattering of only the reactor gives a smooth destructive background symmetric around \((0,0)\) and range given by \(D_{1-1}\) shown in figure 16.

At the same time as having helped in clarifying the physics of the rescattering formulation, (61) provides an easier way to calculate the probe weight function than that outlined in the preceding sections. It will certainly be preferred in future numerical calculations. And could give a useful check of the numeric results shown in section 4.5.

### 4.3.2 Symmetries of distortion prescription

The symmetry

\[
\langle j_1m_1 j_2m_2 | j_3m_3 \rangle = (-)^{j_1-m_1} \left(\frac{2j_3 + 1}{2j_1 + 1}\right)^{\frac{1}{2}} \langle j_1m_1 j_3 - m_3 | j_2 - m_2 \rangle
\]
of the vector addition coefficients leads to the symmetries

\begin{align}
\langle -M | S(q_{ex}, b_1) | -\mu_1 \rangle &= (-)^{M-\mu_1} \langle \mu_1 | S(q_{ex}, b_1) | M \rangle \\
\langle -M | f(q_{ex}, \delta) | -\mu_1 \rangle &= (-)^{M-\mu_1} \langle \mu_1 | f(q_{ex}, \delta) | M \rangle
\end{align}

(65) (66)

for the effective formfactor and the probe weight function.

Further, when the distortion functions \( D_1(b_1) \) and \( D_2(b_2) \) are assumed to depend on the absolute value of the impact parameter only, as is the case in the eikonal absorption limit described in section 4.3, the symmetries

\begin{align}
\langle \mu_1 | S(q_{ex}, \tilde{b}_1) | M \rangle &= \langle -\mu_1 | S(q_{ex}, b_1) | -M \rangle \\
\langle \mu_1 | f(q_{ex}, \tilde{\delta}) | M \rangle &= \langle -\mu_1 | f(q_{ex}, \delta) | -M \rangle
\end{align}

(67) (68)

are satisfied. Here \( b_1 = (b_X, b_Y) \) and \( \tilde{b}_1 = (b_X, -b_Y) \). In deriving this the symmetry

\[ \langle j_1 m_1 j_2 m_2 | j_3 m_3 \rangle = (-)^{j_1+j_2-j_3} \langle j_1-m_1 j_2-m_2 | j_3-m_3 \rangle \]

(69)

of the vector addition coefficients was used.

4.3.3 The slope of the polarization response

Some calculations of the individual amplitudes with absorption, but ignoring the rotation from the NN to the external system were done. This helped in finding the explanation of the slope of the polarization response over the quasi elastic region as seen in the calculations presented in section 4.5.

It turns out that the slope appears already in the cross-section-weighted sum of the polarization response for the individual amplitudes. Since the amplitudes individually give the PW response also when distortion is included (remember rotation NN \( \rightarrow \beta \) frame ignored), the slope comes from the \( \omega \) dependence of the absorption factor. The \( \omega \)-dependence of the absorption factor is rather different for the \( \beta \) and \( \delta \) amplitudes. The explanation is very simple: At small \( \omega \) the response function prefers the smaller effective momentum transfers, at large \( \omega \) the response function prefers the larger effective momentum transfers. The absorption comes (essentially) from the interference between the PW and RS (rescattering) terms, where the PW is evaluated at the \( q_{ex} \) and RS is evaluated at \( q < q_{ex} \) for the smaller \( \omega \) and at \( q > q_{ex} \) for the larger \( \omega \). The absorption factor therefore becomes an increasing function of \( \omega \) for amplitudes whose magnitude is a decreasing function of \( q \), and similarly the absorption factor is a decreasing function of \( \omega \) for amplitudes whose magnitude is an increasing function of the momentum transfer.

The rotation from NN CM frame to the external frame and the interference effects between amplitudes affect the size of the polarization response, but only little its slope. As mentioned earlier, the rotation of the amplitudes as given by (43) leads to different effect of distortion in these two classes of amplitudes: The first three of (42) are not transformed, whereas the latter have their phases modified. The former are somewhat more absorbed than the latter.
4.4 Response function

The nuclear spin isospin response function contains the physics we are trying to address experimentally. The response is defined as

\[ R_{\tau \nu}^{\nu' \sigma' \sigma} (q, q', \omega) = \sum_{n \neq 0} \langle 0 | \hat{j}_{\tau \nu} \sigma_{\mu} (q) | n \rangle \delta (\omega - (E_n - E_0)) \langle n | \hat{j}_{\nu' \sigma'}^{\dagger} (q') | 0 \rangle \]  

(70)

with the spin-isospin current given by

\[ \hat{j}_{\tau \nu} \sigma_{\mu} (q) = i \sum_{j=1}^{A} \tau_{\tau \nu}^{j} \sigma_{\sigma_{\mu}}^{j} e^{-iq \cdot \tau_{j}} \]  

(71)

\[ \sigma_{00} = 1, \quad \sigma_{10} = \sigma_{z}, \quad \sigma_{11} = \mp \frac{1}{\sqrt{2}} (\sigma_{x} \pm i \sigma_{y}) \]

\[ \tau_{00} = 1, \quad \tau_{10} = \tau_{z}, \quad \tau_{11} = \mp \frac{1}{\sqrt{2}} (\tau_{x} \pm i \tau_{y}) \]

Upper spin operator indices \( \mu' \) of the response function transform as bra's and lower spin indices \( \mu \) transform as ket's under spatial rotation. The \( \delta \) function selects the energy transferred to the nucleus.

The nuclear states \(| n \rangle \) entering in (70) are the eigenstates of the Hamiltonian

\[ H = H_{sp} + V \]  

(72)

including the residual interaction \( V \). If the particle-hole interaction \( V \) is turned off, the response is referred to as uncorrelated, i.e. the pure shell model response. One may say that the validity of the approximative methods used for the calculation of the correlated response is the question we are addressing experimentally.

The self consistent response was calculated within the continuum RPA framework as described in [43]. The effective particle hole interaction was parametrized as

\[ V(q, \omega) = \left( \frac{f_{\pi}}{m_{\pi}} \right)^{2} \left\{ \left( g' - \frac{q^{2}}{m_{\pi}^{2} - t} \Gamma_{\pi}^{2} (t) \right) (\sigma^{1} \cdot \hat{q})(\sigma^{2} \cdot \hat{q}) \right. \]

\[ + \left. \left( g' - C_{\rho} \frac{q^{2}}{m_{\rho}^{2} - t} \Gamma_{\rho}^{2} (t) \right) (\sigma^{1} \times \hat{q})(\sigma^{2} \times \hat{q}) \right\} \tau^{1} \cdot \tau^{2} \]  

(73)

with \( m_{\pi} = 139 \text{ MeV} \), \( m_{\rho} = 770 \text{ MeV} \), \( C_{\rho} = 2.18 \) and \( f_{\pi} = 1.008 \). Cut-off masses in the monopole form factors \( \Gamma_{\pi} \) and \( \Gamma_{\rho} \) were 1300 MeV and 2000 MeV respectively [33, 43]. Coupling to the \( \Delta \) with coupling constant \( f_{\pi}^{*} = 2.0 f_{\pi} \) was included in the calculation. The parameter \( g' \), which determines the interaction at zero momentum transfer, was in calculations shown in this manuscript set to 0.6. It is emphasized that (73) should be regarded as a parametrization, rather than a model of the interaction. The main feature of interest for us, is the contrast between the attractive longitudinal and the repulsive transverse piece of \( V \) at large momentum transfers. As can be seen from the plane wave calculations presented in section 4.5, the softening and enhancement of the longitudinal response versus the hardening and quenching of the transverse
response leads to a preference for longitudinal response in the lower end of the quasi elastic peak: The polarization response $P$ predicted in PWIA becomes an increasing function of the energy transfer over the quasi free region.

In order to calculate the cross section and polarization response of the $(d, 2p)$ reaction, the traces

$$\text{tr}(\mathcal{M} \tau_{\lambda \mu} \mathcal{M}^\dagger) = 2 \times \left( \frac{m_1 m_2}{m_1 m_2} \right)^2 \times$$

$$\sum_{M'M'} \langle 1M' | \tau_{\lambda \mu} | 1M \rangle \sum_{\mu_1 \sigma_1 \mu_2 \sigma_2} \frac{d^3 \delta}{(2\pi)^2} \int \frac{d^3 \delta'}{(2\pi)^2} R_{\tau \nu_1 \nu_2}^{\mu_1 \mu_2} (q_{ex} - \delta, q_{ex} - \delta', \omega)$$

$$\langle \mu_1 | f(q_{ex}, \delta) | M \rangle \star t_{\mu_1 \sigma_1 \mu_2 \sigma_2} (q_{ex} - \delta) \langle \mu_1' | f(q_{ex}, \delta') | M' \rangle t_{\mu_1' \sigma_1' \mu_2' \sigma_2'} (q_{ex} - \delta')$$

are needed (see A.1). (Do not confuse the $\tau_{\lambda \mu}$ acting in the spin space of the deuteron with the isospin transition operator in (71).)

The cross section is

$$\frac{d^2 \sigma}{dt \, d\omega} = \frac{1}{364 \pi F^2} \text{tr}(\mathcal{M} \mathcal{M}^\dagger)$$

$$F^2 = p_1 p_2 - (m_1 m_2)^2 = |p_{1\text{lab}}|^2 m_2^2$$

$F$ is the incoming flux and the factor $1/3$ averages over initial deuteron spin states. The spherical components of the tensor analyzing powers are

$$T_{\lambda \mu} = \frac{\text{tr}(\mathcal{M} \tau_{\lambda \mu} \mathcal{M}^\dagger)}{\text{tr}(\mathcal{M} \mathcal{M}^\dagger)}$$

### 4.4.1 Local density approximation

A response function calculated in semiclassical approximation will be used for illustration of the physical content of the response. However, in the calculation including absorption, the off-diagonal matrix elements are important. Thus, as illustrated by the deuteron response function in section 4.4.2, the correct treatment of the antisymmetrization is essential. Therefore the quantum mechanical calculation will be used for these calculations.

A simple derivation of the uncorrelated semi classical response function can be made by assuming locally plane waves for the nuclear single particle states.

$$R(q, q', \omega) = \sum_{ph} \langle \hat{0} | e^{i \mathbf{q} \cdot \mathbf{r}} | ph^{-1} \rangle \delta(\omega - (E_p - E_h)) \langle ph^{-1} | e^{i \mathbf{q} \cdot \mathbf{r}} | \hat{0} \rangle$$

$$= \int d^3 r d^3 r' \sum_{p \geq p_0 \left( \frac{\epsilon_p}{\epsilon_h} \right)} \sum_{\epsilon < p_0 \left( \frac{\epsilon_p}{\epsilon_h} \right)} e^{-i(\mathbf{h} + \mathbf{q} - \mathbf{p}) \cdot \mathbf{r}} e^{i(\mathbf{h} + \mathbf{q}' - \mathbf{p}) \cdot \mathbf{r}' \delta(\omega - (\epsilon_p - \epsilon_h))$$

The semi-classical response is rotationally invariant. For a nucleus with equal number of protons and neutrons, ignoring the Coulomb interaction, it is also rotationally invariant in
Figure 17: The off-diagonality function $X(x)$ for a step density distribution in local density approximation.

isospin space. Further the spin and scalar responses are equal, so we need not specify these indices. Thus, in the semiclassical limit, the normalization (70-71) is very natural.

The substitutions $r \rightarrow r + s/2$ and $r' \rightarrow r - s/2$ lead to

$$R(q, q'; \omega) = \int d^3 r \, d^3 s \sum_{p \sim p_F(r)} e^{-i(q - q') \cdot r} e^{-i((q + q')/2) \cdot s} e^{i(h - p) \cdot s} \delta(\omega - (\varepsilon_p - \varepsilon_h))$$

$$= \int d^3 r \, e^{-i(q - q') \cdot r} R\left(\frac{q + q'}{2}, \omega; p_F(r)\right)$$

Equation (78) is nothing but the volume integral of the response density and is therefore often referred to as "local density approximation".

$p_F(r)$ entering in (78) is the semi-classical Fermi momentum for the doubly degenerate Fermi gas given by

$$p_F(r) = \left(2 m_N \varepsilon_F(r)\right)^{1/2} = \left(2 m_N (\lambda - V(r))\right)^{1/2}$$

$$V(r) = V_0 \left(1 + \exp\left(\frac{r - R_0}{a}\right)\right)^{-1}$$
Figure 18: The off diagonality for the semi-classical response function (dashed) on the carbon target is compared with that of the uncorrelated spin transverse quantum mechanical response (full). In the qm calculation the angle between the average momentum transfer and the difference between the momentum transfers was chosen to $\pi/2$; the off diagonality depends, but very little, on this angle for the spin response. (The scalar response, not shown, depends rather strongly on the angle.) In the semiclassical there is no angle dependence. On the left is shown the off diagonality at the quasi elastic peak; here the semiclassical and quantum mechanical off diagonalities are not very different. On the right the off diagonality at energy transfer corresponding to the edge of the quasi elastic peak is shown; here the semiclassical off diagonality is broader than the quantum mechanical, thus leading to an overestimation of the distortion effects in the semiclassical calculation.

The parameters for the Woods-Saxon potential used for the $^{12}$C nucleus are $a = 0.67$ fm (surface thickness), $R_0 = 1.27$ fm $A^{1/3} = 2.91$ fm, and $V_0 = -62.70$ MeV $= 0.318$ fm$^{-1}$. For the $^{40}$Ca the values used are $V_0 = -50$ MeV $= -0.253$ fm$^{-1}$ and $R_0 = 4.34$ fm.

The condition

$$\rho_{\text{s.c.}}(r) = \frac{2}{3\pi^2} p_{\text{F}}^3(r) = \frac{2}{3\pi^2} (2m_N (\lambda - V(r)))^{3/2}$$

(81)

then fixes the Fermi surface at $\lambda = -17.60$ MeV $= -0.0892$ fm$^{-1}$ which, as expected, lies close to the proton separation energy $-15.96$ MeV $= -0.0809$ fm$^{-1}$.

With constant density out to radius $R = (9\pi A/8)^{1/3}/p_\text{F}$ the response can be calculated
Figure 19: The uncorrelated semiclassical response function for $^{12}\text{C}(n,p)$-like reaction in the quasifree region is compared with the full calculation [43] at momentum transfer 1.25 fm$^{-1}$ and 1.75 fm$^{-1}$. The splitting of the full calculation indicates the separation of the spin longitudinal and the spin transverse response due to the spin-orbit interaction in the non-spin-saturated nucleus. The energy in the semi classical calculation is shifted by the energy 13.4 MeV which is the mass difference between $^{12}\text{C}$ and $^{12}\text{B}$. The position of the quantum mechanically calculated response, has been shifted 5 MeV to give the position of the $J^* = 4^-$ state in $^{12}\text{B}$ at $\omega = 18$ MeV.

analytically

$$R(q, q', \omega) = \frac{4\pi}{3} R^3 \mathcal{X}(|q - q'| R) R \left( \frac{q + q'}{2}, \omega; p_F \right)$$  \hspace{1cm} (83)$$

$$\mathcal{X}(x) = \frac{3}{x^2} \left( \frac{\sin x}{x} - \cos x \right)$$  \hspace{1cm} (84)$$

Ignoring Pauli blocking, the diagonal infinite nuclear matter response $\text{Vol} \times R(q, \omega, p_F)$ is a parabola as a function of $\omega$. It peaks with a value of $R_{\text{max}} = (3/4)(A m_s)/(q p_F)$ at the energy transfer $\omega = q^2/2m_s$, and intersects the axis at $\omega = q^2/2m_s \pm q p_F/m_s$. Pauli blocking gives in the limit in the prescription (78) a linear cut of the response at small energy transfer. From figure 19 it is seen, that this general structure holds also for the response integrated as in (78). Here the comparison with the quantum mechanical calculation shows that, at low momentum transfers, the Pauli blocking is underestimated in the local density approximation.

The off-diagonality function $\mathcal{X}$ is plotted in figure 17. The first zero crossing lies at $x \simeq 4.5$. Thus the off-diagonality of the response is of order $4.5 p_F \left(8/(9\pi A)\right)^{1/3} (\simeq 1.4 \text{ fm}^{-1}$ for the $^{12}\text{C}$ nucleus). The off-diagonality of the response is a volume effect thus scaling as the inverse of the nuclear radius. The off-diagonality of the response function is of the same order of magnitude as the distorting momentum transfer $\delta$. Since the distortion effects are large, it is therefore
important to have a good description of the off-diagonality. A comparison between the off-
diagonality given by the semiclassical description and the full quantum mechanical is given
in figure 18. As can be see from (58) and (74), there is a phase space factor in the integral
(74) scaling like |q_{ex} - q| for the interference between the plane wave and rescattering terms.
And proportional to |q - q'|^2 for the pure rescattering term. The difference seen in the figure
between the semi classical and quantum off diagonality thus gives rise to an overestimation of
distortion effects in the semi-classical calculation. Away from the peak even negative estimates
of cross section can occur.

4.4.2 Response on reference targets

The proton and deuteron targets serve as reference as well as test targets. Furthermore the
proton target, i.e. the p(d, 2p)n reaction, has been used to normalize the reaction cross section.

The angular distribution on the proton and deuteron targets, i.e. dσ/dt for p(d, 2p)n and
d(d, 2p)2n is found to follow the plane wave prediction very accurately out to momentum
transfers of √-t = 2.4 fm⁻¹, that is over three orders of magnitude (see figure 4).

The charge exchange response on the proton target is particularly simple

\[ R_{11\mu}(q, q') = R_{00\mu}(q, q') = 2 \quad (85) \]

resulting from δ-function like wavefunctions and with the isospin factor |(p | τ- | n)|^2 = 2
included.

The response on the deuteron target gives a good feeling for the properties of the response
functions. The off-diagonality is very sensitive to the antisymmetrization of the two-neutron
final state. This can also expected to be the case for the more general nuclear target.

The charge exchange spin response function for the deuteron target may be calculated
assuming plane waves for the final state. For the 1S0 final state of the two-neutron system this
is a rather crude approximation, but this final state contributes only little to the total response.
Only at strictly forward scattering, it would be important to treat the 1S0 of the di-neutron
state carefully.

Only the response integrated over energy transfer ω will be calculated. The general expres-
sion for the response on the deuteron target is taken as

\[ R_{σμσ′μ′}(q, q') = \frac{1}{3} \sum \langle d | \exp(-i(q \cdot r) \frac{σ_{μμ′}}{2}) \sigma_{σμ} \tau_- | 2n \rangle \langle 2n | \exp(i(q' \cdot r) \frac{σ_{μ′μ}}{2}) \sigma_{σμ}^{\dagger} \tau_+ | d \rangle \quad (86) \]

The sum is over initial and final states. Some details of the calculation are outlined in appendix
B. The scalar response in which only the triplet final state contributes is found to be

\[ R_{00\mu}(q, q') = 2 \int j_0(|q - q'|^2) U^2(r) + W^2(r) \, dr \]

\[ - \int j_0(|q + q'|^2) U^2(r) + W^2(r) \, dr \quad (87) \]

The direct contribution of the spin response is

\[ R_{11\mu}(q, q') = 2(\frac{1}{3} + \frac{2}{3}) \int j_0(|q - q'|^2) U^2(r) + W^2(r) \, dr \quad (88) \]
The separation in the first line corresponds to $1/3$ two-neutron singlet contribution and $2/3$ triplet contribution. The quantization axis is along $(q - q')$. The sum (89) is rotationally invariant, so we are free to change co-ordinate system when adding the exchange contribution. Quantized along $(q + q')$ this has the same expression, except, the triplet part with opposite sign and $q'$ replaced by $-q'$. The total response function is then given by

\[ R^\mu_\nu(q, q') = 2 \int j_0(|q - q'|^2/2) (U^2(r) + W^2(r)) \, dr - 2 \frac{1}{3} \int j_0(|q + q'|^2/2) (U^2(r) + W^2(r)) \, dr - 2 \frac{2 - |\mu|}{3} \int j_2(|q + q'|^2/2) W^2(r) \, dr \]  

(90)

which is no longer isotropic.

The direct term is identical to the semi classical result, and identical for spin and scalar responses. The antisymmetrization brings in extra terms which break the rotational invariance of the spin response. At large momentum transfers the antisymmetrization becomes less important in the diagonal response $R(q, q)$, and the deuteron response reduces to the proton response function.

The response on the deuteron target gives us the simplest example of why the semiclassical approximation fails: It does not include properly the antisymmetrization of the nuclear states. This is important when large off-diagonal components of the response are of significance for the result, i.e. when absorption effects are large. It is therefore reasonable to say that on heavier nuclei than the deuteron it can be expected that the use of the semiclassical response gives misleading results.

### 4.5 Experimental results

In this section experimental results obtained in the quasifree region for the $^{12}$C and $^{40}$Ca targets are presented. The data are compared with the plane wave and distorted wave calculations with and without correlations in the nuclear response calculated in the continuum RPA scheme.

Considering first the measured differential cross sections in the laboratory frame, we observe a well formed quasi elastic peak as high as hundreds of $\mu$barn/sr/MeV at forward angles. The experimental differential cross section on $^{12}$C and $^{40}$Ca are the same apart from a factor: the position and width of quasi elastic peak is practically identical at similar momentum transfers.

The peak appears at higher energy transfer than the plane wave prediction at the smallest momentum transfers, figures 20 and 21 corresponding to momentum transfers 0.7 fm$^{-1}$ and 1.3 fm$^{-1}$ respectively. At higher momentum transfer the peak appears to have strength moved below the PW calculation, see figures 22 to 24. The “dispersion relation” corresponds rather to
transferring the energy and momentum on two nucleons than on one. The naive picture where one nucleon in the target with momentum $p$ is struck gives an energy transfer

$$\omega = \frac{q^2}{2m_N} + \frac{p \cdot q}{m_N} \quad (91)$$

If the momentum transfer is distributed equally on two nucleons the peak occurs at half of this energy transfer. This is likely to be the case at large scattering angles. However, at small scattering angles, the direct and the rescattering momentum transfer could have significant non-parallel components, allowing the energy transfer to exceed that of $91$.

We saw, in the discussion on the elastic distortion function in section 4.3.1, that the rescattering distributes the momentum transfer on the two probe nucleons. This is also clear intuitively, since, experimentally, we require the two protons to be in a quasi-bound state. The same argument would apply to the case of inelastic rescattering. Thus, when the energy transfer is high enough that the final state Pauli blocking is not important, a large component of cross section corresponding to this process may occur. At smaller energy transfer the two step inelastic process is highly blocked and its cross section is expected to describe only a minor fraction of the observed. This may explain why the distortion calculation, which only accounts for rescattering in the elastic channel, strongly underestimates the cross section at large energy transfers.

The questions addressed here are closely linked to the experimental difference between $(p, p')$ and $(p, n)$ reactions on nuclei. In the iso-scalar channel, dominated by the spin-scalar amplitude, the nucleon-nucleon amplitudes fall off slower with momentum transfer than in the isospin channel. This makes inelastic rescattering relatively less favorable for the $(p, p')$ than for the $(p, n)$ reaction. This may explain why, in $^{12}$C$(p, p')$ the cross section, even at momentum transfer as large as $1.7 \text{fm}^{-1}$ is reproduced in a DWBA calculation; whereas in a calculation of $^{12}$C$(p, n)$ only half of the experimental cross section is found in the DWBA calculation [43].

At the smallest scattering angles, $2.7^\circ$ and $5.0^\circ$, the polarization response at the peak of the quasi elastic bump is in fair agreement with the free represented by the deuteron target if figure 4, though with a slight tendency towards more longitudinal response. At larger scattering angles, the measured response is systematically much more spin transverse than on the deuteron target. An enhancement, though smaller, of the spin transverse over the spin longitudinal cross section in nuclei was observed by J. McClelland and collaborators using the $(p', p'')$ reaction [35, 36] and more recently in $(\vec{p}, \vec{n})$ [11].

Neither the PW calculation nor the DW calculation show a significant effect of correlations for the position of the quasi elastic peak. The spin longitudinal component is moved downwards in energy transfer relative to the spin transverse which is moved a bit upwards. But the total cross section remains practically unchanged both in magnitude and position. Thus we find no support for the hypothesis that the altered dispersion relation on the nuclear target could be due to a medium polarization.

In the PW prediction the softening of the spin longitudinal mode shows up only in the polarization response. In particular, note that at momentum transfer $1.7 \text{fm}^{-1}$, figure 22, the critical momentum for pion condensation, where the effect is expected to be strongest, we observe a significant enhancement of the spin longitudinal response at small energy transfers. At higher energy transfer the effect of correlation is small.
Turning to the DW calculation. Here two effects occur: First, and most important, the distortion mixes the spin longitudinal and spin transverse channel. Secondly, due to the fact that the reaction is somewhat surface peaked, thus selecting lower densities, the effect of nuclear correlations are damped. Both effects damp the difference between correlated and uncorrelated response at the level of the observed quantity $P$. In figures 20 - 24 we see that, in the continuum region, there is only very little difference between the correlated and the uncorrelated polarization response.

The comments above concerning the absorption calculation are mainly based on the results on the carbon target, for which the results at small energy transfers are in fair agreement with the data. On the $^{40}$Ca target the absorption factor is of order 1/10. This means that the distortion effects become very sensitive to the shape radial shape of the distortion functions $D_1(b_1)$ and $D_2(b_2)$. In order to reproduce the experimental cross section at low energy transfers we would need distortion functions of 15-20% larger range. And even on the carbon target it may be necessary to reconsider how to assign the radial structure of the distortion functions. A procedure where measured optical potentials, integrated along the beam-ejectile trajectory replaces the distortion functions, is under consideration.

In figure 28 calculations with the spectator distortion reduced a factor 3, and with spectator absorption turned off are shown. With no distortion of the spectator, the cross section is proportional to the plane wave cross section, and the polarization response is not far from the PW. When the spectator distortion is turned on, cross section is reduced mainly in the region of large energy transfers. But at the same time the spin directions at low energy transfers are reorganized, bringing the response closer to the measured. With full distortion we have already seen that the response agrees qualitatively with the data at low energy transfers.

The lesson: it seems necessary to include inelastic rescattering in the description to get a reasonable description of the spin channel of the measured nuclear response. Until having "found" the missing cross section we should be very cautious in interpreting the experimental data, on the spin variables exciting the nucleus to the quasifree region in $(\vec{p}, \vec{p'})$, $(\vec{p}, \vec{n})$ and $(\vec{d}, 2p[1S_0])$ reactions, as indicative of polarization effects at the level of the nuclear response function.
Figure 20: Cross section and tensor polarization response as measured in $^{12}\text{C}(d,2p\;[^1S_0])$ at laboratory bombarding energy 1600 MeV and 2.7° scattering angle. The momentum transfer is around 0.7 fm$^{-1}$. The curves on the right plot are PW CRPA (full) PW C0TH (dash), the calculated PW cross sections have been divided by 4. The curves on the left plot are DW CRPA (full) DW C0TH (dash).
Figure 21: Cross section and tensor polarization response as measured in $^{12}\text{C}(d,2p[^1S_0])$ at laboratory bombarding energy 1600 MeV and 5.0° scattering angle. The momentum transfer is around 1.3 fm$^{-1}$. The curves on the right plot are PW CRPA (full) PW C0TH (dash), the calculated PW cross sections have been divided by 4. The curves on the left plot are DW CRPA (full) DW C0TH (dash).
Figure 22: Cross section and tensor polarization response as measured in $^{12}\text{C}(\vec{d}, 2p) [^1S_0]$ at laboratory bombarding energy 1600 MeV and 6.5° scattering angle. The momentum transfer is around 1.7 fm$^{-1}$. The curves on the right plot are PW CRPA (full) PW C0TH (dash), the calculated PW cross sections have been divided by 4. The curves on the left plot are DW CRPA (full) DW C0TH (dash).
Figure 23: Cross section and tensor polarization response as measured in $^{12}\text{C}(\vec{d},2p[^1S_0])$ at laboratory bombarding energy 1600 MeV and 8.0° scattering angle. The momentum transfer is around 2.1 fm$^{-1}$. The curves on the right plot are PW CRPA (full) PW C0TH (dash), the calculated PW cross sections have been divided by 4. The curves on the left plot are DW CRPA (full) DW C0TH (dash).
Figure 24: Cross section and tensor polarization response as measured in $^{12}\text{C}(d,2p[^1S_0])$ at laboratory bombarding energy 1600 MeV and 9.3° scattering angle. The momentum transfer is around 2.4 fm$^{-1}$. The curves on the right plot are PW CRPA (full) PW C0TH (dash), the calculated PW cross sections have been divided by 4. The curves on the left plot are DW CRPA (full) DW C0TH (dash).
Figure 25: Cross section and tensor polarization response as measured in $^{40}\text{Ca}(\vec{d},2p)\,^{1S_0}$ at laboratory bombarding energy 2000 MeV and 4.3° scattering angle. The momentum transfer is around 1.3 fm$^{-1}$. The curves on the right plot are PW CRPA (full) PW C0TH (dash), the calculated PW cross sections have been divided by 10. The curves on the left plot are DW CRPA (full) DW C0TH (dash).
Figure 26: Cross section and tensor polarization response as measured in $^{40}\text{Ca}(\vec{d}, 2p)^{[1S_0]}$ at laboratory bombarding energy 2000 MeV and 5.7° scattering angle. The momentum transfer is around 1.7 fm$^{-1}$. The curves on the right plot are PW CRPA (full) PW COTH (dash), the calculated PW cross sections have been divided by 10. The curves on the left plot are DW CRPA (full) DW COTH (dash).
Figure 27: Cross section and tensor polarization response as measured in $^{40}\text{Ca}(\vec{d},2p\,^1S_0)$ at laboratory bombarding energy 2000 MeV and 7.2$^\circ$ scattering angle. The momentum transfer is around 2.1 fm$^{-1}$. The curves on the right plot are PW CRPA (full) PW COTH (dash), the calculated PW cross sections have been divided by 10. The curves on the left plot are DW CRPA (full) DW COTH (dash).
Figure 28: The PW (dash) is compared with the DW when the distortion parameter $\gamma$ for the spectator is reduced a factor 3 (full), and with no absorption on the spectator (dots). The cross section for PW calculation is divided by 4. Both calculations are with uncorrelated nuclear response function.
5 Epilogue

Focus has been put on the "elementary" nucleon-nucleon interaction in the spin-isospin channel. Some guidance was found in describing this interaction in terms of pion exchange with a small impact parameter cut. Apart from being interesting in itself, this is of interest since the interaction serves as the driving force in the investigation of the nuclear spin-isospin response in the $\Delta$ region and the quasi elastic region.

Exciting the $\Delta$ in the nucleus $^{12}$C, the $(d,2p[^1S_0])$ reaction has proven to produce surprising results: On the carbon target, the ratio $(T/L)$ of spin transverse to spin longitudinal cross section is significantly larger than on the deuteron target. The result on the nuclear target should be taken with some caution: Distortion effects on the tensor analyzing powers, which have not yet been estimated for the $\Delta$ region, are expected to be large.

In the quasi elastic region the result is similar: Whenever the response on nuclei deviates from the free response, it is observed to be relatively more spin transverse. Here estimates of the effect of distortion have been made in the eikonal limit, but allowing for only one inelastic scattering. Effects of absorption are found to be large, bringing the observed tensor analyzing power closer to the observed. In regions where the cross section is reproduced, i.e. at small energy transfers, also the observed analyzing power is in qualitative agreement with the calculation. Distortion effects seem to wash out the effect of a possible medium polarization, leaving less effect, if any, of correlations at the level of the measured quantity. At energy transfers comparable to the Fermi energy, only a minor fraction of the observed cross section is reproduced by the calculation. We are therefore not yet able to draw firm conclusions from the measured polarization response in this region.

6 Thanks

First of all, I want to thank the French-Scandinavian collaboration as a member of which I have been working the latest years. The members of the team are D. Bachelier, J. L. Boyard, A. Brockstedt, C. Ellegaard, C. Gaarde, C. Goodman, T. Hennino, J. C. Jourdain, J. S. Larsen, M. Österlund, P. Radvanyi, B. Ramstein, M. Roy-Stephan, and P. Zupranski. It is the collective effort of these people that has resulted in the experimental data presented in this work. I have found it inspiring to work in a group not bigger than leaving the possibility of recognizing the importance of each individual, and yet big enough to host most different entrances to physics.

M. Ichimuras stay for a semester in Lund three years ago initiated a fruitful work with him and K. Kawahigachi on the application of their method for the calculation of the self consistent continuum response to the $(d,2p)$ reaction. This collaboration is very much appreciated.

V. Dmitriev has contributed in an important way to our understanding of distortion effects occurring when using composite projectiles as probes for nuclear spin isospin modes, in bringing the attention to the importance of taking into account the distortion for the projectile nucleons separately. On this problem, as on validity of the semi classical response function, we have benefitted from discussions with V. Zelevinski too.

H. Bech Nielsen has been involved in the continuing work on the the poor mans absorption model applied to the one-pion-exchange mechanism. It is my hope that an understanding of
the short range piece of the interaction will emerge from this stimulating collaboration.

From the \((\vec{p}, \vec{n})\) group at Los Alamos, J. McClelland, T. Carey and T. Taddeuchi have been most important for the development of the spin-isospin physics at high momentum transfers. Their early \((\vec{p}, \vec{p}')\) work in many ways initiated the field. Continuous discussions with them at conferences and during visits at Los Alamos are very helpful in the common effort to resolve the puzzle which was first seen in their \((\vec{p}, \vec{p}')\) experiment in the quasi elastic region [35]: Why does the transverse cross section appear enhanced over the spin longitudinal in spin isospin excitations of nuclei?

The contact with C. Wilkin and D. V. Bugg has been very helpful during the work with the \((d, 2p)\) reaction. As is clear from the reference list, they have been very active in the calculation of the \((d, 2p)\) reaction. In particular their experience with the calculation of the reaction on the elementary targets has been valuable to us.

It is a pleasure to thank J. Delorme and P. Guichon for discussions on their semiclassical calculation of the \((^3\text{He}, t)\) on nuclei in the \(\Delta\) region. – And F. Osterfeldt for discussions on his fully quantum mechanical calculation, with T. Udagawa and S. W. Hong [10], of the \((^3\text{He}, t)\) and \((d, 2p)\) reactions in both the quasi elastic and the delta region. – Discussions with G. Chanfray on the possibility of surface effects mixing the spin longitudinal response with the spin transverse in the semi classical calculation are acknowledged. – Furthermore we have benefitted from stimulating discussions with R. D. Smith on the double scattering formulation in momentum representation. – Among many other physicists whose comments we have benefitted from, I want to mention P. Arve, G. Bertsch and H. Esbensen.

Carlsberg Fondet, som har afholdt min løn gennem tre år ved NBI, og som har ydet støtte til deltagelse i konferencer, takkes for den imødekommende behandling af mine ansøgninger.

A Description of polarized particles

The notation to describe an ensemble of polarized particles is developed. Analyzing powers in spherical notation appropriate to describe polarization phenomena for particles with spin 1 or higher are defined.

A brief summary of the transformation properties of helicity states is given. It is based on discussions which may be found in the literature [31, 45, 46]. It is mainly given to be able derive the transformations used when coupling the spin of the reactive projectile nucleon in \( p(d,2p)\Delta^0 \) to the spectator nucleon in the on-shell approximation for the \( N + N \rightarrow N + \Delta \) reaction as given in A.3.

The experimental techniques to obtain polarized particles will not be discussed here. I refer to the discussion by A. Boudard [47] for a brief discussion of the polarized ion source at LNS.

A.1 The spin density operator and the analyzing powers

An ensemble is a classical admixture of quantum mechanical systems. The individual quantum system is described by a state vector

\[
| i \rangle = \sum_i c_m(i) | \alpha_i, sm \rangle
\]

(92)

\[
c_m(i) = \langle sm | i \rangle
\]

here expanded on spin eigenstates. It occurs in the ensemble with (classical statistical) weight \( w(i) \). Consider for example a beam of particles prepared in different spin states. \( w(i) \) is the number of particles prepared in the state \( | i \rangle \) when the total number of particles is normalized to 1.

The polarization properties of the ensemble are contained in the spin density operator

\[
\rho = \sum_i | i \rangle w(i) \langle i |
\]

(93)

\[
= \frac{1}{2s + 1} \sum_{\lambda\mu} \rho_{\lambda\mu} \tau^\dagger_{\lambda\mu}
\]

(94)

\[
\rho_{\lambda\mu} = \text{trace}(\rho \tau_{\lambda\mu})
\]

(95)

which is positive semi-definite with trace 1. \( \lambda \) takes integer values \( \lambda = 0, 1, ..., 2s \), and \( \mu = -\lambda, ..., \lambda \). The fundamental tensor operator \( \tau \) acting in the spin space of the polarized particle is normalized as

\[
\langle sm'| \tau_{\lambda\mu} | sm \rangle = \sqrt{2\lambda + 1} \langle sm \lambda\mu | sm' \rangle
\]

(96)

Since the expectation value of a given operator, \( O \), can be calculated when the density operator is known

\[
\langle O \rangle = \sum_i w(i) \langle i | O | i \rangle
\]

(97)

\[
= \text{trace}(\rho O)
\]

(98)
it contains all information on the polarization of the ensemble.

Next, consider an experiment where a beam of polarized particles are scattered of a target. Let $\mathcal{M}$ be the scattering matrix for the reaction. It is then convenient to define analyzing powers

$$T_{\lambda\mu} = \frac{\text{tr}(\mathcal{M} \tau_{\lambda\mu} \mathcal{M}^\dagger)}{\text{tr}(\mathcal{M} \mathcal{M}^\dagger)}$$

(99)

where the trace is taken over spins of the outgoing particles. The cross section corresponding to a beam with spin density given by (94) is then

$$d\sigma = (d\sigma)_0 \sum_{\lambda\mu} \rho_{\lambda\mu} T_{\lambda\mu}^\dagger$$

(100)

The restriction from parity conservation of strong interactions in the scattering process $1 + 2 \rightarrow 3 + 4$ is expressed through

$$S = \sum_{i=1}^{4} S_i = e^{i\pi S_\pi} \mathcal{M} e^{-i\pi S_\pi} \pi_1 \pi_2 \pi_3 \pi_4$$

(101)

$S$ is the sum of the spin operator of all the particles and $\pi_i$ denotes the intrinsic parity of a particle [48]. With the $y$-axis along the normal to the reaction plane $\hat{n}$ this implies the symmetry relation

$$T_{\lambda\mu} = (-)^{\lambda-\mu} T_{-\lambda-\mu}$$

(102)

for the tensor analyzing powers. This leaves only $T_{20}$, $T_{21}$, and $T_{22}$ as independent rank two analyzing powers. In the present manuscript the $y$-axis is chosen along $\hat{n}$. This has the advantage that the rotations are represented by the real matrices $d_{\mu\beta}^{\lambda}$.

In the experiment at Saturne it has been possible to determine only a combination of $T_{20}^{M}$ and $T_{22}^{M}$ of the form

$$P = \rho_{20}^{\text{beam}} T_{20}^{N}$$

$$= \rho_{20}^{\text{beam}} \left( \frac{1}{2} T_{20}^{M} + \sqrt{\frac{3}{2}} \cos 2\varphi T_{22}^{M} \right)$$

$$\simeq \rho_{20}^{\text{beam}} \left( \frac{1}{2} T_{20}^{M} + \sqrt{\frac{3}{2}} (\cos 2\varphi) T_{22}^{M} \right)$$

(103)

In the first line both $T_{\lambda\mu}$ and $\rho_{20}^{\text{beam}}$ had common quantization axis along the normal to the symmetry axis of the synchrotron. In the following lines the analyzing powers refer to the Madison frame [49] defined in section 2.

The transformation between the cartesian tensor analyzing powers and the spherical analyzing powers goes as

$$A_{XX} = -\sqrt{2} \left( \frac{1}{2} T_{20} - \sqrt{\frac{3}{2}} T_{22} \right)$$

$$A_{YY} = -\sqrt{2} \left( \frac{1}{2} T_{20} + \sqrt{\frac{3}{2}} T_{22} \right)$$

(104)

(105)
A.2 Helicity representation

A helicity state of a physical system in a frame $K$ is defined from a Lorentz boost $L_z(\zeta)$ of rapidity $\zeta$ along the $z$-axis of a reference state $|\bar{p}sh\rangle$ followed by a rotation $R(\theta)$ of the state into the direction of the motion

$$|psh\rangle = \mathcal{H}(\zeta)|\bar{p}sh\rangle$$  \hspace{1cm} (106)

$$\mathcal{H}(\zeta) = R(\theta)L_z(\zeta)$$  \hspace{1cm} (107)

For massive particles the reference four momentum may be taken as $\vec{p} = (0,i m)$.

Let $K'$ be a co-ordinate system moving with rapidity $-\Sigma$ in $K$. The boost $L$ connects the momenta in the two frames

$$|p'\rangle_{K'} = L(\Sigma)|p'\rangle_K$$  \hspace{1cm} (108)

$$|p'\rangle_K = L(\Sigma)|p\rangle_K$$  \hspace{1cm} (109)

The transformation of the helicity state defined by (106) is then

$$L(\Sigma)|psh\rangle = L(\Sigma)\mathcal{H}(\zeta)|\bar{p}sh\rangle = \mathcal{H}(\zeta')|\tilde{p}s\rangle$$  \hspace{1cm} (110)

Operating from the left with $\mathcal{H}(\zeta')^{-1}$ the unspecified spin state $|\tilde{p}s\rangle$ is found

$$|\tilde{p}s\rangle = \mathcal{H}(\zeta')^{-1}L(\Sigma)\mathcal{H}(\zeta)|\bar{p}sh\rangle$$  \hspace{1cm} (111)

Since

$$R(\chi) = \mathcal{H}(\zeta')^{-1}L(\Sigma)\mathcal{H}(\zeta)$$  \hspace{1cm} (112)
leaves \( \vec{p} \) unchanged it is a rotation in the spin space. The boosted state may be written
\[
\mathcal{L}(\Sigma) \left| p \, s \, h \right\rangle = \sum_{h'} D_{h'h}^\dagger(\mathcal{R}) \left| p' \, s' \, h' \right\rangle
\]
(113)
Or, expressing the state vector in \( \mathcal{K}' \) in terms of the states in \( \mathcal{K} \)
\[
\left| s' \, h' \right\rangle_{\mathcal{K}'} = \sum_{h} D_{h'h}^\dagger(\mathcal{R}) \left| s \, h \right\rangle_{\mathcal{K}}
\]
\[
= \sum_{h} d_{h'h}^\dagger(\chi) \left| s \, h \right\rangle_{\mathcal{K}}
\]
(114)
In the last line advantage has been taken of the special choice of axes indicated in figure 29: \( \mathcal{L}(\Sigma) \) along the z-axis, and momenta in the \( xx \) plane.\(^2\)

The equation (112) may be rewritten
\[
\mathcal{L}_z(\zeta') \mathcal{R}_y(\chi) \mathcal{L}_z^{-1}(\zeta) = \mathcal{R}_y^{-1}(\theta') \mathcal{L}_z(\Sigma) \mathcal{R}_y(\theta)
\]
(115)
In the Galilean limit \( \chi \approx \theta - \theta' \), the difference is known as the Thomas precession.

A general Lorentz transformation may be written
\[
\mathcal{G} = \mathcal{R}(\chi) \mathcal{L}(\zeta) = \exp(-i \frac{\chi}{2} \sigma) \exp(-i \frac{\zeta}{2} \cdot \sigma)
\]
(116)
in a representation by Hermitian \( 2 \times 2 \) matrices of trace unity [46]. The special cases in need here are a boost of rapidity \( \zeta \) along the z-axis and a rotation the angle \( \chi \) around the y-axis
\[
\mathcal{L}_z(\zeta) = \exp(-i \frac{\zeta}{2} \sigma_z) = \cosh \frac{\zeta}{2} - \sigma_z \sinh \frac{\zeta}{2}
\]
(117)
\[
\mathcal{R}_y(\chi) = \exp(-i \frac{\chi}{2} \sigma_y) = \cos \frac{\chi}{2} - i \sigma_y \sin \frac{\chi}{2}
\]
(118)
with the identification
\[
cosh \zeta = \gamma = \text{Lorentz factor of boost}
\]
\[
sinh \zeta = \gamma \beta
\]
Inserting (117) and (118) in (115) and equating the elements proportional to 1, \( \sigma_x, \sigma_y, \) and \( \sigma_z \) leads to the equations
\[
\cos \frac{\chi}{2} \cosh \frac{\zeta}{2} \cos \frac{\zeta'}{2} - \cos \frac{\chi}{2} \sinh \frac{\zeta}{2} \sinh \frac{\zeta'}{2} = \cosh \frac{\Sigma}{2} \sin \frac{\theta}{2} \sin \frac{\theta'}{2} + \cos \frac{\Sigma}{2} \cos \frac{\theta}{2} \cos \frac{\theta'}{2}
\]
\[
\sin \frac{\chi}{2} \cosh \frac{\zeta}{2} \sinh \frac{\zeta'}{2} + \sin \frac{\chi}{2} \sin \frac{\zeta}{2} \cosh \frac{\zeta'}{2} = \sinh \frac{\Sigma}{2} \cos \frac{\theta}{2} \sin \frac{\theta'}{2} + \sin \frac{\Sigma}{2} \sin \frac{\theta}{2} \cos \frac{\theta'}{2}
\]
\[
\sin \frac{\chi}{2} \sin \frac{\zeta}{2} \sinh \frac{\zeta'}{2} + \sin \frac{\chi}{2} \sin \frac{\zeta}{2} \cos \frac{\zeta'}{2} = \cosh \frac{\Sigma}{2} \sin \frac{\theta}{2} \cos \frac{\theta'}{2} - \sin \frac{\Sigma}{2} \cos \frac{\theta}{2} \sin \frac{\theta'}{2}
\]
\[
\cos \frac{\chi}{2} \sin \frac{\zeta}{2} \cos \frac{\zeta'}{2} - \cos \frac{\chi}{2} \cosh \frac{\zeta}{2} \sinh \frac{\zeta'}{2} = \sin \frac{\Sigma}{2} \sin \frac{\theta}{2} \sin \frac{\theta'}{2} - \sin \frac{\Sigma}{2} \cos \frac{\theta}{2} \cos \frac{\theta'}{2}
\]

\(^2\)The convention for the rotation matrices is that used by Bohr and Mottelson [50]. Jacob and Wick [31] and by Martin and Spearman [45] use the complex conjugate, the (real) \( d \)-functions coincide.
The sum of the squares of these four equations gives

$$\cosh \Sigma = \cosh \zeta \cosh \zeta' - \sinh \zeta \sinh \zeta' \cos \chi$$  \hspace{1cm} (119)

$$\cos \chi = \frac{\cosh \zeta \cosh \zeta' - \cosh \Sigma}{\sinh \zeta \sinh \zeta'}$$  \hspace{1cm} (120)

This is the cosine relation for the hyperbolic geometry which is used for all the relativistic transformations done in the following.

### A.3 Transformations used in calculations of $p(\bar{d}, 2p)\Delta^0$

The Breit frame of the probe is defined to be the co-ordinate system in which the beam and ejectile particles have opposite momenta of equal size. Thus, it is the CM-frame of the beam-ejectile system, and provides the best place to calculate the $(d, 2p)$ formfactor with nonrelativistic wave functions.

The invariant energy squared can be expressed either in terms of the incoming or the outgoing particles

$$s = (E_1 + E_2)^2 - (p_1 + p_2)^2$$  \hspace{1cm} (121)

$$s = (E_3 + E_4)^2 - (p_3 + p_4)^2$$  \hspace{1cm} (122)

Adding these expressions gives

$$2s = \sum_{i=1}^{4} m_i^2 + 2 E_1 E_2 + 2 E_3 E_4 - 2 p_1 \cdot p_2 - 2 p_3 \cdot p_4$$  \hspace{1cm} (123)

In the Breit frame where $E_1 = E_3 = (m^2 - t/4)^{1/2}$, $m =$"probe mass", $p_1 = -p_3 = q/2$, and $E_2 = E_4$ this may be rewritten as

$$2s = 2m^2 + m_2^2 + m_4^2 + 2(4m^2 - t)^{1/2}E_{24}^n - t$$  \hspace{1cm} (124)

or

$$E_{24}^n = \frac{t + 2s - (2m^2 + m_2^2 + m_4^2)}{2(4m^2 - t)^{1/2}}$$  \hspace{1cm} (125)

In the on-shell approximation where the elementary amplitudes, representing the NN $\rightarrow$ NN or NN $\rightarrow$ NΔ transition, are taken as free amplitudes $M(s, t)$ one is forced to let the kinematics of these amplitudes refer to a higher bombarding energy per nucleon than available in the probe. The expression (125) can be used to determine the bombarding energy which the underlying nucleon-nucleon collision should refer to.

In the impulse approximation the average momentum of the spectator particle in the probe vanishes. Thus, on average, the Breit frame of the projectile-ejectile system (the $(d, 2p)$ probe) coincides with the Breit frame of the elementary projectile-ejectile (the underlying $(n, p)$ probe). This means, that $E_{24}^n$ can be determined from (125), and then $s_{NN}$ can be calculated from (124) with a single nucleon mass inserted instead of the probe mass $m$

$$2s_{NN} = 2m_N^2 + m_2^2 + m_4^2 + 2(4m_N^2 - t)^{1/2}E_{24}^n - t$$  \hspace{1cm} (126)
Table 1: Kinematics for (d, 2p) at $T_{\text{lab}} = 2000 \text{MeV}$ and $\theta_{\text{lab}} = 4.3^\circ$ on nucleon target. $T_{NN}$ grows rather quickly with $m_4$. The angle $\chi$ is close to $90^\circ$ close to elastic kinematics where momentum transfer is essentially perpendicular to the beam and ejectile momenta, and closer to $0^\circ$ in the highly inelastic kinematics for the $\Delta$ region where the momentum transfer has a substantial component along the beam.

In general this procedure leads to the result that the bombarding energy in the underlying $(n, p)$ reaction is higher than the bombarding energy per nucleon for the composite probe. In table 1 a numeric example is given.

A second application of the expression (125) for the energy of particle 2 concerns the transformation from the Breit frame to the Madison frame.

By convention the Madison frame is the laboratory frame with $z$ axis along the beam momentum and with $y$-axis along $p_\perp = p_x \times p_z$ [15]. For the beam particle the representation of the spin states coincides with the laboratory helicity representation.

The energy of particle 2 (the target) is therefore just its mass in the Madison frame, and in the Breit frame it is given by (125). This leads to the Lorentz factor

$$\cosh \Sigma^{B\rightarrow M} = \frac{E_2^M}{m_2} = \frac{t + 2s - (2m^2 + m_4^2 + m_2^2)}{2m_2(4m^2 - t)^{1/2}}$$

(127)

for the transformation. Since also the two other sides

$$E_1^M = \frac{s - m_2^2 - m_2^2}{2m_2}$$

(128)

$$|p_1^M|^2 = -\frac{t}{4}$$

(129)

of the triangle on the hyperbolic surface that characterizes the transformation of particle 1 are known, it is possible to write down the cosine relation of the rotation between the helicity representation in the two frames (see (114) and (119))

$$\cosh \Sigma^{B\rightarrow M} = \cosh \zeta_1^M \cosh \zeta_1^B - \sinh \zeta_1^M \sinh \zeta_1^B \cos \chi$$

(130)

with the notation

$$\gamma_i = \cosh \zeta_i = \frac{E_i}{m_i}, \quad \gamma_i \beta_i = \sinh \zeta_i = \frac{|p_i|}{m_i}$$

(131)
Solving for the rotation of the helicity from the Madison frame to the Breit frame of the probe (around the $\hat{n} = \hat{p}_1 \times \hat{p}_3\cdot$axis) yields
\[
\chi = \arccos \frac{2m^2 (m_2^2 - m_3^2) - t (s + m^2 - m_3^2)}{(-t(4m^2 - t)((s - m^2 - m_3^2)^2 - 4m^2m_3^2))^{1/2}}
\] (132)

where it is emphasized that $m$ is the mass of the composite probe (the average of the deuteron and di-proton masses).

Next, consider the transformation from the nucleon–nucleon CM frame to the Breit frame. This transformation is of practical importance since the short range correlation effects are calculated in the CM frame of the “fundamental” scattering process. And we need to transform to the Breit frame to multiply the (direction dependent) formfactor of the probe on the matrix elements. In figure 30 the geometry of the problem is shown. The transformation again involves the techniques for transforming helicity states, four rotations, i. e. one for each of the incoming and one for each of the outgoing particles. Expressing the transformation in the spin state by the $D$-functions
\[
c_M(h' | h)_B = D_{h'h'}^*(0, \chi, 0) = d_{h'h'}^*(\chi)
\]

the helicity amplitudes in the Breit frame may be written
\[
\langle h_3 h_4 | \mathcal{M}(s, t) | h_1 h_2 \rangle_B = \sum_{h_1' h_2' h_3' h_4'} d_{h_1'h_1'}^*(\chi_1) d_{h_2'h_2'}^*(\chi_2) d_{h_3'h_3'}^*(\chi_3) d_{h_4'h_4'}^*(\chi_4) c_M(h_3'h_4' | \mathcal{M}(s, t) | h_1'h_2')_C_M
\]

The rapidity $\zeta$ of the Breit frame seen from the nucleon–nucleon CM frame is obtained from the cosine relations for the angles between the direction of the coordinate transformation and the probe momenta in the Breit frame ($\psi$ and $(\pi - \psi)$ indicated in figure 30)
\[
cosh \zeta_{1}^{CM} = \cosh \zeta_{1}^{B} \cosh \zeta - \sinh \zeta_{1}^{B} \sinh \zeta \cos \psi
\]
\[
cosh \zeta_{3}^{CM} = \cosh \zeta_{3}^{B} \cosh \zeta - \sinh \zeta_{3}^{B} \sinh \zeta \cos (\pi - \psi)
\]

Since $\zeta_{1}^{B} = \zeta_{3}^{B}$ these equations add to
\[
cosh \zeta_{1}^{CM} + \cosh \zeta_{3}^{CM} = 2 \cosh \zeta_{13}^{B} \cosh \zeta
\] (137)

or
\[
cosh \zeta = \gamma_{13}^{CM-B}
\]
\[
= \frac{\cosh \zeta_{1}^{CM} + \cosh \zeta_{3}^{CM}}{2 \cosh \zeta_{13}^{B}}
\]
\[
= \frac{E_1^{CM} + E_3^{CM}}{2 E_{13}^{B}}
\]
\[
= \frac{2s_{NN} + 2m_4^2 - m_2^2 - m_0^2}{2(s_{NN}(4m_3^2 - t))^{1/2}}
\] (138)
The geometry involved in the transformation, with Lorentz $\gamma$-factor $\cosh \zeta$ along the dashed line, of helicity states going from the CM frame of the "elementary" reaction to the Breit frame of the probe. In the Breit frame the beam and ejectile nucleon have opposite momenta. $\theta$ is the CM scattering angle.

where the insertion of

$$E_1^{CM} = \frac{s_{NN} + m_1^2 - m_2^2}{2\sqrt{s_{NN}}} \quad E_3^{CM} = \frac{s_{NN} + m_2^2 - m_1^2}{2\sqrt{s_{NN}}}$$

and (129) helped in getting to the last line.

The cosine relation of the hyperbolic geometry gives the relations ($i = 1, \ldots, 4$)

$$\cosh \zeta = \cosh \zeta_i^{CM} \cosh \zeta_i^{B} - \sinh \zeta_i^{CM} \sinh \zeta_i^{B} \cos \chi_i$$

which results in

$$\chi_i = \pm \arccos \frac{\cosh \zeta_i^{CM} \cosh \zeta_i^{B} - \cosh \zeta_i}{\sinh \zeta_i^{CM} \sinh \zeta_i^{B}} \pm, \quad i = 1, 4$$

All quantities on the right hand side are given above. Direct insertion does not make calculation much easier than numeric calculation of each term. Only the probe rotations give fairly nice formulæ, which are formally identical to (132)

$$\chi_1 = \arccos \frac{(s_{NN} + m_1^2 - m_2^2) t - 2 m_1^2 (m_2^2 - m_1^2)}{[-t (s_{NN} - (m_1 + m_2)^2) (s_{NN} - (m_1 - m_2)^2) (4 m_1^2 - t)]^{1/2}}$$

$$\chi_3 = -\arccos \frac{(s_{NN} + m_1^2 - m_2^2) t + 2 m_1^2 (m_2^2 - m_1^2)}{[-t (s_{NN} - (m_1 + m_4)^2) (s_{NN} - (m_1 - m_2)^2) (4 m_3^2 - t)]^{1/2}}$$
In the plane wave description on a single nucleon only these two rotations are needed. The helicities of the target and daughter nucleon or $\Delta$ are summed over, so the rotations of these helicities do not appear in the result.

In the example in table 1 we see that, not surprisingly, the rotations $\chi_1$ and $\chi$ almost cancel. This means that if we had no spin dependence for the $(d,2p)$ formfactor, it would be a good approximation to ignore the rotation. However, as we saw in section 2.2, there is a strong spin dependence, thus making it essential to include the rotations.

Note that in the Breit frame helicity representation particle 1 is quantized along $q$ (parallel to $p_1$) and particle 3 along $-q$. Thus a rotation of $\pi$ around the $y$-axis of the spin-index for the ejectile nucleon is done before coupling it to the outgoing spectator proton.

On nuclear target in the quasi elastic region it is, for target and daughter nucleus rather the transformation from the NN CM frame to the Breit frame of the 2-4 system which is needed. These transformations are again given by (142) and (143). However, at energies as high as 800 MeV per nucleon, the effect of the transformations are only small and have not been performed. Also, I will not enter into the discussion on how to make a (maybe) better approximation for unbound final states (often referred to as optimum frame approximation) which is a subject of current investigation [51, 52].

B Calculation of deuteron response function

The response function for the deuteron target writes

$$R_{\sigma'\mu'}^{\mu}(q, q') = \frac{1}{3} \sum \int \frac{d^3 k}{(2\pi)^3} d \mathbf{r} d \mathbf{r}'$$

$$\langle l m_l 1 m_s | 1 M \rangle \frac{U_l(r)}{r} Y_{l m_l}^*(\mathbf{r}) \left\{ (\frac{1}{2}) 1 m_s | \sigma_{\sigma'\mu} | (\frac{1}{2}) s m \right\}$$

$$\exp(-i q \cdot \frac{\mathbf{r}}{2}) \frac{1}{\sqrt{2}} \left( e^{i \mathbf{k} \cdot \mathbf{r}} + (-)^s e^{-i \mathbf{k} \cdot \mathbf{r}} \right) \frac{1}{\sqrt{2}} \left( e^{-i \mathbf{k} \cdot \mathbf{r}'} + (-)^s e^{i \mathbf{k} \cdot \mathbf{r}'} \right) \exp(i q' \cdot \frac{\mathbf{r}'}{2})$$

$$\langle (\frac{1}{2}) s m | \sigma_{\sigma'\mu'}^t | (\frac{1}{2}) 1 m_s' \rangle \frac{U_{\nu}(r')}{r'} Y_{\nu m_l}^t(\mathbf{r}') \langle l' m_l' 1 m_s' | 1 M \rangle \quad (144)$$

The isospin factor $|\langle p | \sigma_- | n \rangle|^2 = 2$ is included. The $\frac{1}{\sqrt{2}}$-normalization of the two-neutron final state corresponds to normalization to $d\mathbf{k}$-integration over $4\pi$ (running through the final state phase space twice).

Integration over $d^3 k$ in (144) leads to $\delta$-functions $\delta(\mathbf{r}' \pm \mathbf{r})$ which brings the expression on the form

$$R_{\sigma'\mu'}^{\mu}(q, q') = 2\frac{1}{3} \sum \int d \mathbf{r} \left( \exp(i \frac{\mathbf{q} - \mathbf{q}'}{2} \cdot \mathbf{r}) + (-)^s \exp(i \frac{\mathbf{q} + \mathbf{q}'}{2} \cdot \mathbf{r}) \right)$$

$$\langle l m_l 1 m_s | 1 M \rangle \frac{U_l(r)}{r} Y_{l m_l}^*(\mathbf{r}) \langle 1 m_s | \sigma_{\sigma'\mu} | s m \rangle$$

$$\langle l' m_l' 1 m_s' | 1 M \rangle \frac{U_{\nu}(r')}{r'} Y_{\nu m_l}^t(\mathbf{r}') \langle s m | \sigma_{\sigma'\mu'}^t | 1 m_s' \rangle \quad (145)$$
The product of the two spherical harmonics is then written

\[ Y_{lm_1}^* (r) Y_{lm_2}^* (r) = \frac{(-)^{m_1}}{4\pi} \left( \frac{2l+1}{2l'+1} \right)^{1/2} \sum_{j=|l-l'|}^{l+l'} \langle l-m_1 l'm_1' j m_1'-m_1 \rangle \]

\[ \langle 000 | j0 \rangle \left( \frac{4\pi}{2j+1} \right)^{1/2} Y_{jm_1'-m_1} (r) \]

and the exponential is expanded on spherical harmonics

\[ e^{i p \cdot r} = 4\pi \sum_{lm} i^l j_l (p r) Y_{lm} (p) Y_{lm}^* (r) \]

(147)

with \( p = (q \pm q')/2 \).

After this, the quantization axis is chosen along the symmetry axis, \( q - q' \) for the direct term and \( q + q' \) for the exchange term. It is then possible to do the summation of all the terms and arrive at the result as given by (87) and (89).

C Pion exchange

The pion nucleon vertex is written

\[ \pi \text{NN} : \quad i g_\pi \bar{u}(p_3, h_3) \gamma_5 u(p_1, h_1) \tau \cdot \pi \]

\[ = \quad ig_\pi \left( \left( \frac{E_1 + m_1}{2m_1} \right) \left( \frac{E_3 + m_3}{2m_3} \right) \right)^{1/2} \left( \frac{1}{2} h_3 \frac{\sigma \cdot p_1}{E_1 + m_1} - \frac{\sigma \cdot p_3}{E_3 + m_3} \frac{1}{2} h_1 \right) \tau \cdot \pi \]

\[ = \quad ig_\pi \left( \left( \frac{E_1 + m_1}{2m_1} \right) \left( \frac{E_3 + m_3}{2m_3} \right) \right)^{1/2} \left( \frac{|p_1| 2 h_1}{E_1 + m_1} - \frac{|p_3| 2 h_3}{E_3 + m_3} \frac{1}{2} h_3 \right) \tau \cdot \pi \]

\[ = \quad ig_\pi \left( \left( \frac{E_1 + m_1}{2m_1} \right) \left( \frac{E_3 + m_3}{2m_3} \right) \right)^{1/2} \left( \frac{|p_1| 2 h_1}{E_1 + m_1} - \frac{|p_3| 2 h_3}{E_3 + m_3} \right) d_{h_1 h_3}^2 (x) \tau \cdot \pi \]

(148)

\[ N.R. : \quad \frac{i g_\pi}{\sqrt{2 m_1 m_3}} \sigma \cdot q \tau \cdot \pi = \frac{i f_\pi}{m_\pi} \sigma \cdot q \tau \cdot \pi \]

(149)

and \( \Delta \) production vertex is

\[ \pi \text{N\Delta} : \quad i \frac{f_\pi}{m_\pi} \bar{u}_\mu(p_4, h_4) u(p_2, h_2) q_\mu T \cdot \pi \]

\[ = \quad i \frac{f_\pi}{m_\pi} \left( \left( \frac{E_2 + m_2}{2m_2} \right) \left( \frac{E_4 + m_4}{2m_4} \right) \right)^{1/2} \]

\[ \left\{ \frac{3}{2} h_4 \left( \left( \frac{E_2 - m_2 \cdot p_4}{m_4 (E_4 + m_4)} \right) S \cdot p_4 - S \cdot p_2 \right) \right\} \left( 1 - \frac{(\sigma \cdot p_4) (\sigma \cdot p_2)}{(E_4 + m_4) (E_2 + m_2)} \right) \frac{1}{2} h_2 \right\} T \cdot \pi \]

65
With these expressions it is fairly straightforward to divide out the \( b_h h' \) from the product of the vertex factors and deduce \( B(s, t) \) of equation (23) needed to make the pole expansion prescribed by the Poor Mans Absorption model.

In the above spin \( \frac{1}{2} \) particles were described by spinors

\[
\begin{align*}
  u(p, m) &= \left( \frac{E + m}{2m} \right)^{\frac{1}{2}} \left( \begin{array}{c} \chi_m \\ \frac{\sigma \cdot p}{E + m} \chi_m \end{array} \right) \\
  \langle u | u \rangle &= \bar{u} u = 1
\end{align*}
\]  

Spin \( \frac{3}{2} \) was described as a coupling between a spin \( \frac{1}{2} \) and a spin 1 spinor

\[
\begin{align*}
  u_\mu(p, m) &= (w_\mu(p, \cdot) u(p, \cdot))(1_{\frac{1}{2}})^{\frac{1}{2} m} \\
  &= \sum_{m', m''} \langle 1m' \frac{1}{2} m'' | \frac{3}{2} m \rangle w_\mu(p, m') u(p, m'') \\
  \langle u | u \rangle &= 1
\end{align*}
\]

with the spin 1 spinor given by

\[
\begin{align*}
  w_\mu(p, m) &= \left( e_m + \frac{p \cdot e_m}{m(E + m)} p, \frac{p \cdot e_m}{m} \right) \\
  \langle w | w \rangle &= w_\mu w_\mu = 1
\end{align*}
\]  

\[
e_m = \begin{cases} 
  \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right), & m = 1 \\
  (0, 0, 1), & m = 0 \\
  \left( \frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}, 0 \right), & m = -1
\end{cases}
\]
Useful formulas in the derivation were

\[(\sigma \cdot p_4) (\sigma \cdot p_2) = p_4 \cdot p_2 + (p_4 \times p_2) \cdot i\sigma = |p_2||p_4|(\cos \theta - \sin \theta i \sigma_\nu)\]  \hspace{1cm} (160)

\[
\langle \frac{3}{2} m_4 | S_\mu | \frac{1}{2} m_2 \rangle = \langle \frac{1}{2} m_2 1 \mu | \frac{3}{2} m_4 \rangle \stackrel{\mu=0}{=} \sqrt{\frac{2}{3}} \delta_{m_2 m_4} \]  \hspace{1cm} (161)
References


[11] J. McClelland and T. Taddeuchi, private communication, 1990, The preliminary analysis of the recent ⁴⁰Ca(¯p, ¯n) experiment performed at momentum transfer around 1.7fm⁻¹ at LAMPF confirms the enhancement of the spin transverse channel over the spin longitudinal seen in (¯p, ¯p').


[19] Loyal Durand III and Yam Tsi Chiu, Phys. Rev. 137B (1965) 1530, Single particle exchange models for the reactions $\pi p \rightarrow \rho p$, $\bar{p} p \rightarrow \bar{Y} Y$, and $n p \rightarrow p n$.


[26] A. B. Wicklund, ANL-HEP-TR-86-21, Argonne National Laboratory, 1986, Data tables for $p p \rightarrow p\pi^+ n$ at 3, 4, 6 and 11.75 GeV/c, see also [27].


[51] R. D. Smith, private communication, 1989, *Valuable discussions on the optimum frame approximation, and on the rescattering problem in (\textsuperscript{3}He, t) and (d, 2p) reactions are appreciated*.