Medical diagnostic ultrasound
- physical principles and imaging

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Preface

This document attempts to introduce the physical principles of medical diagnostic ultrasound to a broad audience ranging from non-engineering students to graduate level students in engineering and science. This is sought achieved by providing chapters with different levels of difficulty:

Chapters with no asterisk can be read by most.

* These chapters are directed towards *bachelor* students in engineering.

** These chapters are directed towards *graduate* students in engineering.

The document can be studied at a given degree of detail without loss of continuation.

This chapter does not consider blood flow imaging with ultrasound, which is treated excellently elsewhere[5].

1 Introduction

Medical diagnostic ultrasound is an imaging modality that makes images showing a slice of the body, so-called tomographic images (tomo = *Gr. tome, to cut* and graphic = *Gr. graphein*, to write). It is a *diagnostic* modality, meaning that it gathers information about the biological medium without modification of any kind[1].

Ultrasound is sound with a frequency *above* the audible range; thus above 20 kHz. Sound is mechanical energy that needs a medium to propagate. Thus, in contrast to electromagnetic waves, it cannot travel in vacuum.

The frequencies normally applied in clinical imaging lies between 1 MHz and 20 MHz. The sound is generated by a transducer that first acts as a loudspeaker sending out an acoustic pulse along a narrow beam in a given direction. The transducer subsequently acts as a microphone in order to record the acoustic echoes generated by the tissue along the path of the emitted pulse. These echoes thus carry information about the acoustic properties of the tissue along the path. The emission of acoustic energy and the recording of the echoes normally take place at the same transducer, in contrast to CT imaging, where the emitter (the X-ray tube) and recorder (the detectors) are located on the opposite side of the patient.

1. To obtain acoustical contact between the transducer and the skin, a small pressure must be applied from the transducer to the skin. In addition to that, ultrasound scanning causes a very small heating of tissue (less than 1°C) and some studies have demonstrated cellular effects under special circumstances.

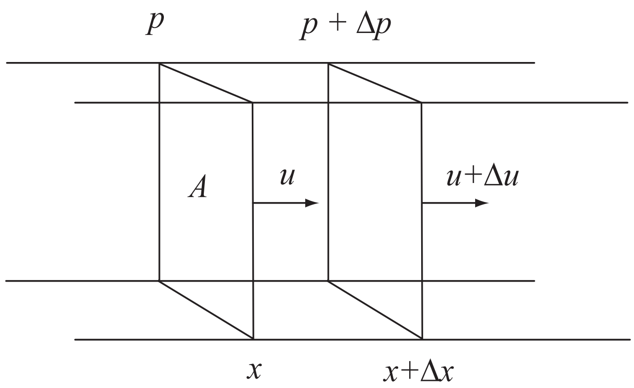

Figure 1 1D situation showing a liquid element inside a sound wave.

This document attempts to give simple insight in to basic ultrasound, simple wave equations, some simple wave types and generation and reception of ultrasound. This is followed by a description of ultrasound's interaction with the medium, which gives rise to the echo information that is used to make images. The different kinds of imaging modalities is next presented, finalized with a description of more advanced techniques. The chapter is concluded with a list of symbols, terms and references.

2 Basics of ultrasound

Ultrasound (as well as sound) needs a medium, in which it can propagate by means of local deformation of the medium. One can think of the medium as being made of small spheres (*e.g.* atoms or molecules), that are connected with springs. When mechanical energy is transmitted through such a medium, the spheres will oscillate around their resting position. Thus, the propagation of sound is due to a continuous interchange between kinetic energy and potential energy, related to the density and the elastic properties of the medium, respectively.

The two simplest waves that can exist in solids are *longitudinal* waves in which the particle movements occur in the same direction as the propagation (or energy flow), and *transversal* (or *shear* waves) in which the movements occur in a plane perpendicular to the propagation direction. In water and soft tissue the waves are mainly longitudinal. The frequency, f, of the particle oscillation is related to the wavelength, λ, and the propagation velocity c:

$$\lambda f = c \tag{1}$$

The sound speed in soft tissue at 37°C is around 1540 m/s, thus at a frequency of 7.5 MHz, the wavelength is 0.2 mm.

2.1 The 1D wave equation*

Describing the wave propagation in 3D space in a lossy inhomogeneous medium ((Danish: *et inhomogent medium med tab*) such as living tissue is very complicated. However, the description in 1D for a homogenous lossless medium is relatively simple as will be shown.

An acoustic wave is normally characterized by its pressure. Thus, in order to obtain a quantitative relation between the particle velocity in the medium, u, and the acoustic pressure, p, a simple situation with 1D propagation in a lossless media will be considered, as shown in Figure 1. This figure shows a volume element of length Δx and with cross-sectional area A. The volume is thus $V = A\Delta x$. The density of the medium - a liquid, for instance, - is ρ and the mass of the element will then be $\rho A\Delta x$.

The pressure p is a function of both x and t. Consider the variation in space first: There will be a pressure difference, Δp, from the front surface at x to the back surface at x+Δx, thus the volume element will be subject to a force $-A\Delta p$. By applying Newton's second law ($F = ma$):

$$-A\Delta p = \rho A\Delta x\frac{du}{dt} \tag{2}$$

or after performing the limit ($\Delta \to$ d)

$$\frac{dp}{dx} = -\rho\frac{du}{dt} \tag{3}$$

Next consider the variations over a time interval Δt. A difference in velocity, Δu, between the front surface (at x) and the back surface (at x+Δx) of the elemental volume will result in a change in that volume which is:

$$\Delta V = A((u + \Delta u)\Delta t - u\Delta t) = A\Delta u\Delta t \tag{4}$$

which in turn is connected with a change in pressure, Δp, according to

$$\Delta V = -\kappa(A\Delta x)\Delta p \tag{5}$$

where κ is the compressibility of the material (*e.g.* a liquid) in units of Pa^{-1}. Performing the same limit as above, gives the second equation:

$$\frac{du}{dx} = -\kappa\frac{dp}{dt} \tag{6}$$

Equations (3) and (6) are the simplest form of the wave equations describing the relation between pressure and particle velocity in a lossless isotropic medium.

3 Types of ultrasound waves

The equations above describe the relation between pressure and displacement of the elements of the medium. Two simple waves fulfilling the above will now be considered. Both are theoretical, since they need an infinitely large medium.

Since optical rays can be visualized directly, and since they behave in a manner somewhat similar to acoustic waves, they can help in understanding reflection, scattering and other phenomena taking place with acoustic waves. Therefore, there will often be made references to optics.

There are two types of waves that are relevant. They can both be visualized in 2D with a square acrylic water tank placed on an overhead projector:

- The *plane* wave which can be observed by shortly lifting one side of the container.
- The *spherical* wave, which can be visualized by letting a drop of water fall into the surface of the water.

When the plane wave is created at one side of the water tank, one will also be able to observe the reflection from the other side of the tank. The wave is reflected exactly as a light beam from a mirror or a billiard ball bouncing off the barrier of the table.

The spherical wave, that on the other hand, originates from a point source and propagates in all directions; it creates a complex pattern when reflected from the four sides of the tank.

3.1 The plane wave*

The *plane wave* is propagating in one direction in space; in a plane *perpendicular* to this direction, the pressure (and all other acoustic parameters) is constant. As a plane extends over the entire space, the plane wave is not physically realizable (but within a given space, an approximation to a plane wave can be obtained *locally*, such as in the shadow of a planar transducer (see later)).

If the plane wave is further restricted to be *monochromatic*, that is, it oscillate at a single frequency, f_0, then the wave equation in 1D is:

$$p(x,t) = P_0 \exp(-\mathrm{j}\,(2\pi f_0 t - 2\pi x/\lambda)) \tag{7}$$

where P_0 is the pressure magnitude (units in pascal, Pa) x is the distance along the propagation direction and $\lambda = c/f_0$. (7) is a complex sinusoid that depends on space and time. The equation will be the same in 3D, provided that the coordinate system is oriented with the x-axis in the propagation direction. The plane wave travelling in the x-direction is sought illustrated by the pressure visualization in Figure 2a.

A plane wave thus propagates in one direction, just like a laser beam, however, it is merely the opposite of a beam.

3.2 The spherical wave*

The other type is a *spherical wave*. It originates from a point (source) and all acoustic parameters are constant at spheres centred on this point. Thus, the equation is the same as in (7), except that the x is substituted with r in a polar coordinate system:

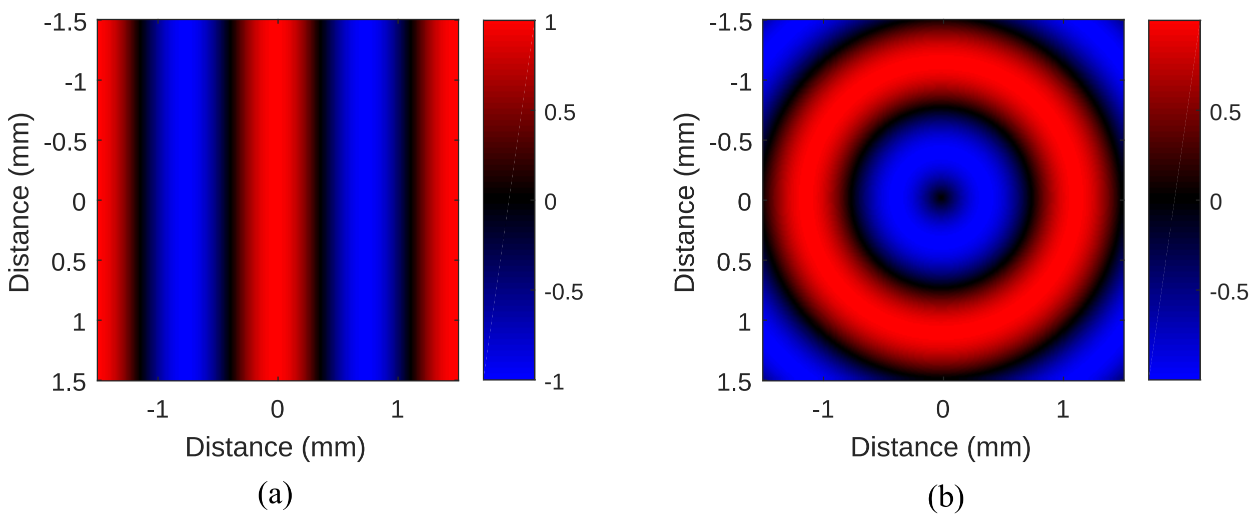

Figure 2 Visualization of 2D plane wave (a) and spherical wave (b) pressure fields at a given instance in time. The pressure fields are monochromatic, *i.e.,* contains only one frequency. Pure black indicates zero pressure, red indicates positive and blue negative pressure values. The wavelength can be read directly from the plots. When including the propagation velocity, c = 1500 m/s, the frequency of the wave can also be found.

Figure 3 Example of modern ultrasound transducer of type 8820e (BK Medical, Denmark) with frequency range 2 - 6 MHz. From www.bkmed.com.

$$p(r,t) = P_0 \exp(-\mathrm{j}\ (2\pi f_0 t - 2\pi r/\lambda)) \tag{8}$$

where r is the distance from the centre of the coordinate system (*i.e.,* the source) to any point in 3D space. The spherical wave are sought illustrated by the pressure visualization in Figure 2b.

Problem 1 With the visualization in Figure 2, measure the wavelength and calculate the centre frequency of the waves.

Problem 2 How should the corresponding 3D plane and spherical waves be understood from Figure 2?

3.3 Diffraction**

An important concept in wave theory is *diffraction*. Ironically, the term diffraction can best be described by what it is *not*: "Any propagating scalar field which experiences a deviation from a rectilinear propagation path, when such deviation is *not* due to reflection or refraction (see later), is generally said to undergo diffraction effects. This description includes the bending of waves around objects in their path. This bending is brought about by the redistribution of energy within the wave front as it passes by an opaque body."[3] Examples where diffraction effects are significant are: Propagation of waves through an aperture in a baffle (*i.e.* a hole in a plate) and radiation from sources of finite size.[3] With the above definition, the only non-diffracted wave is the plane wave.

4 The generation of ultrasound

The ultrasonic transducer is responsible for generating ultrasound and recording the echoes generated by the medium. Since the transducer should make mechanical vibrations in the megahertz range, a material that can vibrate that fast is needed. *Piezoelectric* materials are ideal for this.

The typical transducer consist of a disk-shaped piezoelectric element (the crystal) that is made vibrating by applying an electrical impulse via an electrode on each side of the disc-shaped crystal. Likewise, the echo returning to the crystal makes it vibrate, creating a small electrical potential across the same two electrodes that can be amplified and recorded. In modern clinical scanners, the transducer consists of hundreds of small piezoelectric crystals (or elements) arranged as a 1D array packed into a small enclosure. The shape of this line can be either linear or convex. An example of the latter can be seen in Figure 3. The use of arrays with hundreds of elements, makes it possible to electronically focus and steer the beam, as will be considered later in Chapter 7. However, first the single-element transducer will be considered.

4.1 Piezoelectricity

The acoustic field is generated by using the piezo electric effect present in certain ceramic materials. Electrodes (*e.g.* thin layers of silver) are placed on both sides of a disk of such a material. One side of the disk is fixed to a damping so-called *backing* material, the other side can move freely. If a voltage is applied to the two electrodes, the result will be a physical deformation of the crystal surface, which will make the surroundings in front of the crystal vibrate and thus generate a sound field. If the material is compressed or expanded, as will be the case when an acoustic wave impinges on the surface, the displacement of charge inside the material will cause a voltage change on the electrodes, as illustrated in Figure 4 (left). This is used for emission and reception of acoustic energy, respectively.

4.2 The acoustic field from a disk transducer*

Since the piezoelectric crystal in the ultrasound transducer has a size comparable to or larger than the wavelength, the field generated becomes very complex. Rather than providing equations for describing the field, it will now be attempted visualised.

It is assumed that the piezoelectric, disk-shaped crystal is fixed at the back, as illustrated in Figure 4 (right) and can move freely at the front. Specifically, movement of the surface of the transducer can be described by a velocity vector oriented perpendicular to the surface. In short, the electrical signal applied to the transducer is converted by the electro-mechanical transfer function of the transducer to a velocity function describing the movement of the transducer surface. Note the backing material located behind the crystal; this is used to dampen the free oscillation of the crystal (in the time period just after a voltage is applied), thereby creating a short vibration, when an impulse is applied to the crystal. The radius of the crystal is denoted a. The thickness of the crystal is selected according to the frequency of operation, so that it is $\lambda_{piezo}/2$, where λ_{piezo} is the wavelength of sound in the crystal material.

In order to assess the pressure field generated by the transducer, the surface of the crystal will be divided up into many small surface elements, each contributing to the entire pressure field. If the surface elements are much smaller than the wavelength, they can be considered *point sources*. In the present case, the point source will generate a semi-spherical wave in the space in front of the transducer. The waves are identical, the only difference is the location of the point source. At a given field point in front of the transducer, the total pressure will then be the pressure due to the individual point

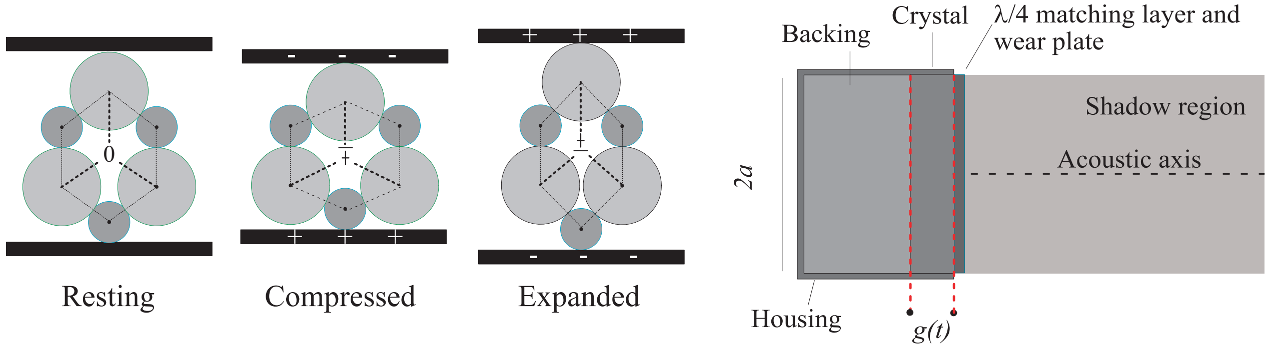

Figure 4 Left: Piezo electric crystal at different states of compression. Right: Single element transducer consisting of piezoelectric crystal with electrodes. This "sandwich" is placed between a backing material and the matching layer towards the medium.

sources. This is an application of Huygens[1] principle. Of course, these individual pressure contributions will interfere positively and negatively dependent on the location of the field point. This interference will result in the final beam, which can be rather complex.

Rather than doing this calculation analytically, a graphical illustration is provided in Figure 5 which shows point sources along a diameter of the transducer disk (the remaining point sources on the disk surface are ignored for simplicity). For each point source, a bow shows the location of the equal-phase-fronts (or equal-time-lag) of the spherical pressure wave generated from that point source at given instances in time. The equal-phase-fronts are not the same as the resulting pressure field; the latter can be obtained by adding the pressure fields of each individual source time-shifted according to the equal-time-lag. Hence, the moving bows in Figure 5 reveal how complicated the field is at a given point.

The "wave" fronts generated by the flat piston transducer in Figure 5 (left) tend towards a (locally) plane wave inside the shadow of the transducer. The pressure field is thus broad, and unsuitable for imaging purposes, as will become clear, when the imaging technique is considered later. In order to focus the ultrasound field and obtain a situation where the acoustic energy travels along a narrow path, a focused transducer is used, as illustrated in Figure 5 (right). In this situation, the individual spherical waves from the transducer are performing constructive interference at the focal point, whereas at all other points, the interference is more or less destructive. In order to make this work efficiently, the wavelength must be much smaller than the distance to the focal point. As an example, a typical depth of the focal point for a 7.5 MHz transducer - 20 mm - will correspond to 100λ.

Notice here, that the key to understand this is the fact that it takes a different amount of time to travel to a given field point from two different source locations. The interference that is caused by this is quite unique for ultrasound.

Example: The interference phenomena can be explored in everyday life: if one positions oneself with one ear pointing into a loudspeaker and turns up the treble, then the sound picture will change if you move in front of the loudspeaker, especially when moving perpendicular to the loudspeaker's acoustic axis. What happens is that the ear is moved to different points in space, which exhibits different amounts of constructive and destructive interference. This phenomenon is less distinct at low frequencies (bass), because the wavelength gets larger. This is also the reason that a stereo sound system can do with one subwoofer for the very low frequency band, but needs two loudspeakers for the remaining higher frequencies.

As noted above, dimensions give most insight, when they are measured in wavelength. Consider the planar transducer in Figure 5 (left): The *near field* from this type of transducer is defined[4] as the region between the transducer and up to a range of a^2/λ. The *far field* region corresponds to field points at ranges *much larger* than a^2/λ. In Figure 5 (left), a is specified, but λ is not. If the transducer frequency is f_0 = 0.5 MHz, then a^2/λ = 33 mm, which is in the middle of the plot. If f_0 = 7.5 MHz, then a^2/λ = 0.5 m! The explanation is as follows: The far field is defined as the region, where there is only moderate to little destructive interference. If this should be possible, then from a given field point in this region, the distance to any point on the transducer surface should *vary* much less than a wavelength: Consider a given field point not on the acoustic axis. Next, draw two lines to the two opposite edges of the transducer. Now the difference in length of these two lines - measured in wavelength - must be much less than one, in order to have little destructive interference at this field point. Thus, the

1. Christian Huygens, physicist from the Netherlands, 1629-95.

higher the frequency, the lower the wavelength, and the farther away one must move from the transducer surface in order to get differences between the length of the two lines much less than one wavelength.

An ultrasound field from a physical transducer will always show a complicated behaviour as can be sensed from Figure 5. Each point source is assumed to emit exactly the same pressure wave (an example of the temporal shape is given in Figure 8). Thus, the circles in Figure 5 indicate spatial and temporal *locations* of each of the individual waveforms. The contribution of all these waveforms would have to be added in order to construct the total pressure field in front of the transducer (however, the circles in Figure 5 only represent point sources on a single diameter across the transducer; many more point sources would be needed to represent the total field from a disk transducer).

Problem 3 Huygens' principle. How would you find - or calculate - how many point sources are needed on the transducer surface in Figure 5 in order to represent the pressure field in front of the transducer with a given accuracy?

Problem 4 Write a short summary of this chapter. This should include the components of the ultrasound transducer: backing layer, crystal and matching layer & wear plate.

5 Ultrasound's interaction with the medium

The interaction between the medium and the ultrasound emitted into the medium can be described by the following phenomena:

The echoes that travel back to the transducer and thus give information about the medium is due to two phenomena: *reflection* and *scattering*. Reflection can be thought of as when a billiard ball bounces off the barrier of the table, where the angle of reflection is identical to the angle of incidence. Scattering (Danish: *spredning*) can be thought of, when one shines strong light on the tip of a needle: light is scattered in all directions. In acoustics, reflection and scattering is taking place when the emitted pulse is travelling through the interface between two media of different acoustic properties, as when hitting the interface of an object with different acoustic properties.

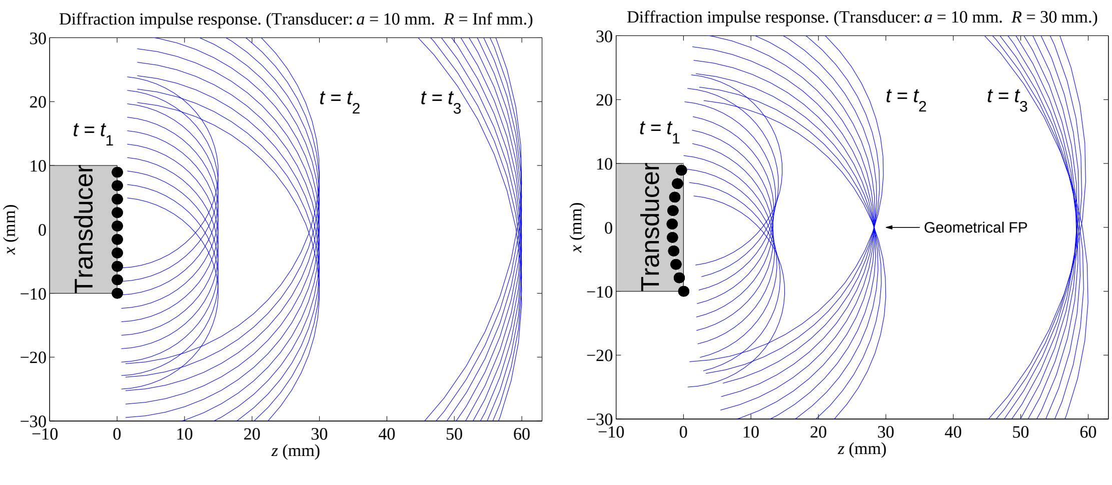

Figure 5 Left: Example of moving circles showing "wave fronts" of equal phase (or equal travel time) at as a function of time from selected point sources (=black dots). For simplicity, only point sources located on a diameter are shown, making this drawing two dimensional. Right: The same for a focused transducer. c = 1500 m/s. The radius of curvature of the disk surface can be deducted from this Figure. What is it?

Specifically, reflection is taking place when the interface is large relative to the wavelength (*e.g.* between blood and intima in a large vessel). Scattering is taking place when the interface is small relative to the wavelength (*e.g.* red blood cell).

The abstraction of a billiard ball is not complete, however: In medical ultrasound, when reflection is taking place, typically only a (small) part of the wave is reflected. The remaining part is *transmitted* through the interface. This transmitted wave will nearly always be *refracted*, thus typically propagating in slightly different direction. The only exception is when the wave impinges perpendicular on a large planar interface: The reflected part of the wave is reflected back in exactly the same direction as it came from (like with a billiard ball) and the refracted wave propagates in the same way as the incident wave.

Reflection and scattering can happen at the same time, for instance, if the larger planar interface is rough. *The more smooth*, the more it resembles pure reflection (if it is completely smooth, *specular reflection* takes place). *The rougher*, the more it resembles scattering.

When the emitted pulse travels through the medium, some of the acoustic (mechanical) energy is converted to heat by a process called *Absorption*. Of course, also the echoes undergo absorption.

Finally, the loss in intensity of the forward propagating acoustic pulse due to reflection, refraction, scattering and absorption is under one named *attenuation*.

5.1 Reflection and transmission*

When a plane wave impinges on a plane, infinitely large, interface between two media of different acoustic properties, *reflection* and *refraction* occurs meaning that part of the wave is reflected and part of the wave is refracted. The wave thus continues its propagation, but in a new direction.

To describe this quantitatively, the *specific acoustic impedance*, z, is introduced. In a homogeneous medium it is defined as the ratio of pressure to particle velocity in a progressing plane wave, and can be shown to be the product of the physical density, ρ, and acoustic propagation velocity c of the medium. Thus, if medium 1 is specified in terms of its physical density, ρ_1, and acoustic propagation velocity c_1, the specific acoustic impedance for this medium is $z_1 = \rho_1 c_1$. The units become kg/(m^{2}s) which is also denoted *rayl*. Likewise for medium 2: $z_2 = \rho_2 c_2$. The interaction of ultrasound with this interface can be illustrated by use of Figure 6, where an incident plane wave is reflected and transmitted at the interface between medium 1 and medium 2. The (pressure) reflection coefficient between the two media is:[2]

$$R = \frac{z_2/(\cos\theta_t) - z_1/(\cos\theta_i)}{z_2/(\cos\theta_t) + z_1/(\cos\theta_i)} \tag{9}$$

where the angle of incidence, θ_i, and transmission, θ_t, are related to the propagation velocities as

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{c_1}{c_2}. \tag{10}$$

Equation (10) is a statement of Snell's law,[2] which also states that: $\theta_r = \theta_i$. The pressure transmission coefficient is $T = 1 + R$.

It should be noted here, that Snell's law applies to optics, where the light can be considered to travel in rays. For a planar wave in acoustics, which only have one direction, the above formulation of Snell's law applies as well. However, when the acoustic wave travels like a beam, Snell's law is only approximately valid. The validity is related to the properties of the beam, namely to which degree the wave field inside the beam can be considered locally plane (which again is related to the thickness of the beam, measured in wavelengths).

Strictly speaking, if the field incident on an interface is not fully planar, and the interaction is to be modelled quantitatively, then the field should be decomposed into a number of plane waves, just like a temporal pulse can be decomposed into a number of infinite tone signals. The plane waves can then be reflected one by one, using (9) and (10).

In the human body, approximate reflection can be observed at the interface between blood and the intima of large vessel walls or at the interface between urine and the bladder wall.

5.2 Critical angle**

Depending on the speed of sound of the two media, some special cases occur.[2]

If $c_1 \geq c_2$, the angle of transmission, θ_t, is real and $\theta_t < \theta_i$, so that the transmitted wave is bent towards the normal to the interface. This can be studied with Figure 6.

If $c_1 < c_2$, the so-called *critical angle* can be defined as

$$\sin\theta_c = \frac{c_1}{c_2}. \tag{11}$$

If $\theta_i < \theta_c$, the situation is the same as above, except that $\theta_t < \theta_i$, *i.e.,* the transmitted wave is bent away from the normal to the interface. This can be studied with Figure 6.

If $\theta_i > \theta_c$, the transmitted wave appear to have a very peculiar form. In short, the incident wave is totally reflected.[2] The interested reader can find more details in larger textbooks[2].

5.3 Scattering*

While reflection takes place at interfaces of infinite size, scattering takes place at small objects with dimensions much smaller than the wavelength. Just as before, the specific acoustic impedance of the small object must be different from the surrounding medium. The scattered wave will be more or less

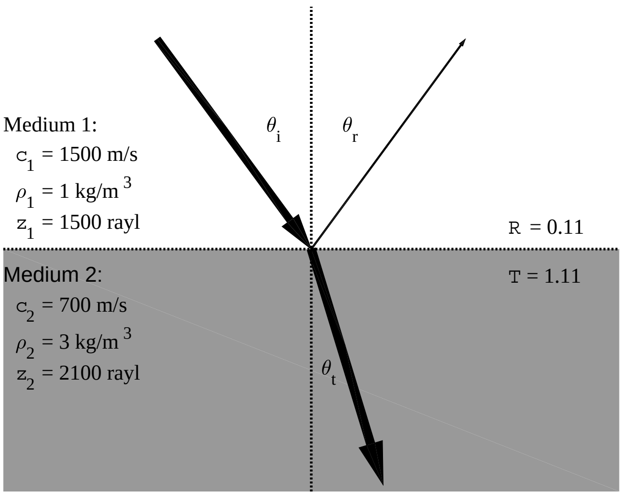

Figure 6 Graphical illustration of Snell's law describing the direction of an incident plane wave (p_i), reflected plane wave (p_r) and transmitted (refracted) plane wave (p_t) from a large smooth interface. The three arrows indicate the propagation *direction* of these three plane waves. The pressure amplitudes of the reflected and transmitted waves are not depicted, but their relative amplitude can be calculated from R and T. $\theta_r = \theta_i$.

spherical, and thus propagate in all directions, including the direction towards the transducer. The latter is denoted *back*scattering.

The scattering from particles much less than a wavelength is normally referred to as *Rayleigh* scattering. The intensity of the scattered wave increases with frequency to the power of four.

Biologically, scattering can be observed in most tissue and especially blood, where the red blood cells are the predominant cells. They have a diameter of about 7 μm, much smaller than the wavelength of clinical ultrasound.

5.4 Absorption*

Absorption is the conversion of acoustic energy into heat. The mechanisms of absorption are not fully understood, but relate, among other things, to the friction loss in the springs, mentioned in Subsection 2. More details on this can be found in the literature.[2]

Pure absorption can be observed by sending ultrasound through a viscous liquid such as oil.

5.5 Attenuation*

The loss of intensity (or energy) of the forward propagating wave due to reflection, refraction, scattering and absorption is denoted attenuation. The intensity is a measure of the power through a given cross-section; thus the units are W/m^2. It can be calculated as the product between particle velocity and pressure: $I = pu = p^2/z$, where z is the specific acoustic impedance of the medium. If $I(0)$ is the intensity of the pressure wave at some reference point in space and $I(x)$ is the intensity at a point x further along the propagation direction then the attenuation of the acoustic pressure wave can be written as:

$$I(x) = I(0)\mathrm{e}^{-\alpha x} \tag{12}$$

where α (in units of m^{-1}) is the attenuation coefficient. α depends on the tissue type (and for some tissue types like muscle, also on the orientation of the tissue fibres) and is approximately proportional with frequency.

As a rule of thumb, the attenuation in biological media is 1 dB/cm/MHz. As an example, consider ultrasound at 7.5 MHz. When a wave at this frequency has travelled 5 cm in tissue, the attenuation will (on average) be 1 dB/cm/MHz x 5 cm x 7.5 MHz = 37.5 dB. For bone, the attenuation is about 30 dB/MHz/cm. If these two attenuation figures are converted to intensity half-length (the distance corresponding to a loss of 50 %) at 2 MHz, it would correspond to 15 mm in soft tissue and 0.5 mm in bone.

Problem 5 Consider a scanning situation, with two interfaces. One located at a depth of 1 cm. There is water between this and the transducer. The other is located at a depth of 2 cm and there is oil from 1 cm to 2 cm. From 2 cm there is water again. The attenuation of water is 0 dB, while it is 1.5 dB/cm/MHz for oil. The transducer frequency is 5 MHz. What is the pressure magnitude at the receiving transducer of the second, relative to the first? (Hint: put the information into a drawing.)

5.6 An example of ultrasound's interaction with biological tissue

When an ultrasound wave travels in a biological medium all the above mechanisms will take place. Reflection and scattering will not take place as two perfectly distinct phenomena, as they were described above. The reason is that the body does not contain completely smooth interfaces of infinite size. And even though the body contain infinitesimally small point objects, the scattered wave from these will be infinitesimally small in amplitude and thereby not measurable!

The scattered wave moving towards the transducer as well as the reflected wave moving towards the transducer will be denoted *the echo* in this document.

So the echo is due to a mixture of reflection and scattering from objects of dimension:

- somewhat larger than the wavelength (example: blood media interface at large blood vessels)
- comparable to the wavelength
- down to maybe a 20th of a wavelength (example: red blood cells).

The effects in Subsection 5.1 - 5.5 are illustrated in Figure 7.

The absorption continuously takes place along the acoustic beam, as media 1 and media 2 (indicated by their specific acoustic impedances) are considered lossy.

Consider the different components of the medium: Scattering from a single inhomogeneity is illustrated at the top of the medium. Below is a more realistic situation where the echoes from many scatterers create an interference signal. If a second identical scattering structure is located below the first, then the interference signal will be roughly identical to the interference signal from the first. The overall amplitude, however, will be a little lower, due to the absorption and the loss due to the first group of scatterers. Notice that the interference signal varies quite a bit in amplitude.

The emitted signal next encounters a thin planar structure, resulting in a well-defined strong echo.

Next, an angled interface is encountered, giving oblique incidence and thus refraction, according to (10) and Figure 6. The change in specific acoustic impedance is the same as above, but due to the non-

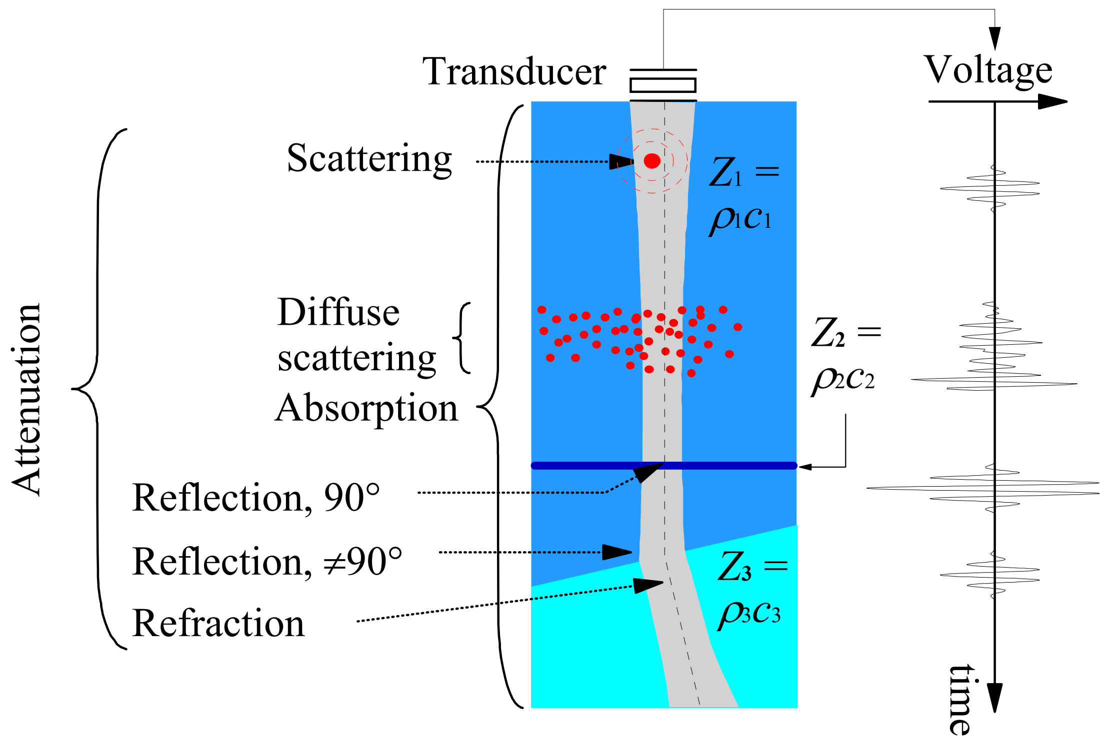

Figure 7 Sketch of the interaction of ultrasound with tissue. The left drawing shows the medium with the transducer on top. The ultrasound beam is shown superimposed onto the medium. The right part of the drawing shows the corresponding received echo signal.

perpendicular incidence, less energy is reflected back. The transmitted wave undergoes refraction, and thus scatterers located below this interface will be imaged geometrically incorrect.

Problem 6 Consider the example in Figure 7: Are the colored regions homogeneous?

6 Imaging

Imaging is based on the *pulse-echo principle*: A short ultrasound pulse is emitted from the transducer. The pulse travels along a beam pointing in a given direction. The echoes generated by the pulse are recorded by the transducer. This electrical signal is always referred to as the received signal. The *later* an echo is received, the *deeper* is the location of the structure giving rise to the echo. The larger the amplitude of the echo received, the larger is the average specific acoustic impedance difference between the structure and the tissue just above. An image is then created by repeating this process with the beam *scanning* the tissue.

All this will now be considered in more detail by considering how **A**mplitude mode, **M**otion mode and **B**rightness mode work.

6.1 A-mode

The basic concept behind medical diagnostic ultrasound is shown in Figure 8, which also shows the simplest mode of operation, A-mode. In the situation in Figure 8 (left) a single point scatterer is located in front of the transducer at depth d. A short pulse is emitted from the transducer, and at time $2d/c$, the echo from the point target is received by the same transducer. Thus, the deeper the point scatterer is positioned, the later the echo from this point scatterer arrives. If many point scatterers (and

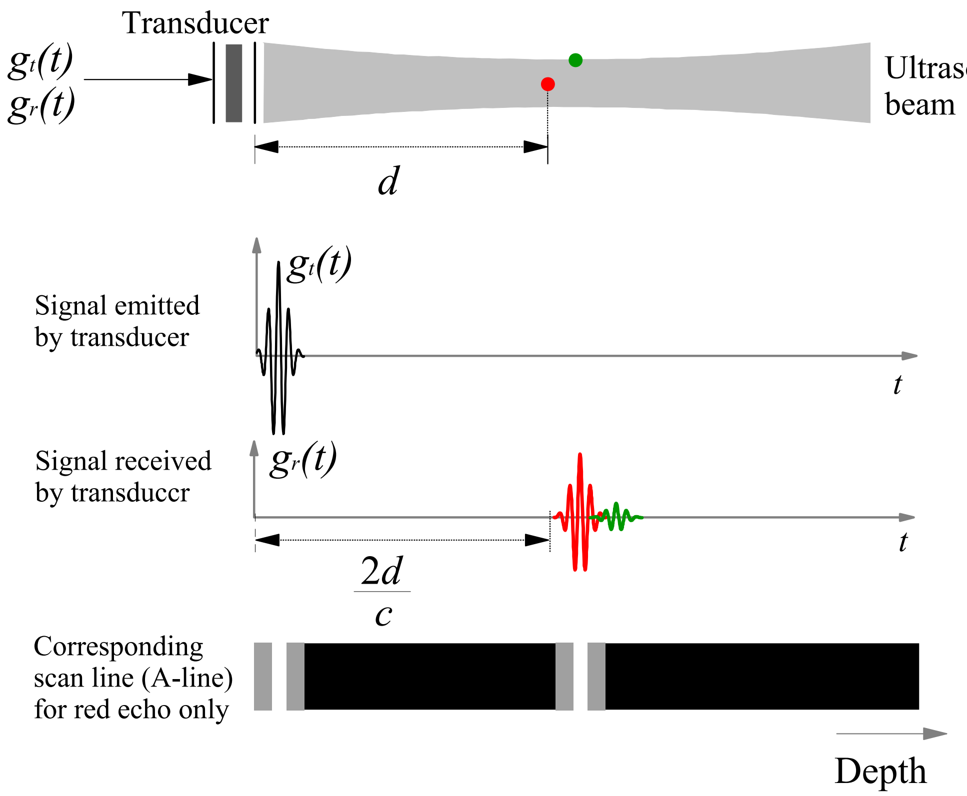

Figure 8 The basic principle behind pulse-echo imaging. An acoustic pulse is emitted from the transducer, scattered by the red point reflector and received after a time interval which is equal to the round trip travel time. The emitted pulse is also present in the received signal due to limitations of the electronics controlling the transducer. The green dot and signal indicates an alternative position yielding a lower echo. Bottom: Estimating the envelope of the received signal followed by calculation of the logarithm yields the scan line in grey scale.

reflectors) are located in front of the transducer, the total echo can be found by simple superposition of each individual echo, as this is a linear system, when the pressure amplitude is sufficiently low.

The *scan line* - shown in Figure 8 lower right - is created by calculating the envelope (*Danish*: indhyllingskurve) of the received signal followed by calculation of the logarithm, in order to compress the range of image values for a better adoption to the human eye. So, the scan line can be called a *gray scale line*. The M-mode and B-mode images are made from scan lines.

6.2 Calculation of the scan line*

The received signal, $g_r(t)$, is Hilbert transformed to $g_{rH}(t)$ in order to create the corresponding analytical signal $\tilde{g}_r(t) = g_r(t) + jg_{rH}(t)$. Twenty times the logarithm of the envelope of this signal, $20\log|\tilde{g}_r(t)|$, is then the envelope in dB, which can be displayed as a gray scale line, as shown in Figure 8 (right). Such a gray scale bar is called a *scan line*, which is also the word used for the imaginary line in tissue, along which $g_r(t)$ is recorded. Note, that because the envelope process is not fully linear, the scanner does not constitute a fully linear system.

Unfortunately, clinical ultrasound scanners do not feature images in dB. More image improvements takes place in the scanner (typically proprietary software) and the gray scale is thus - at best - a pseudo dB-scale, in this document denoted “dB”.

6.3 M-mode

If the sequence of pulse emission and reception is repeated infinitely, and the scan lines are placed next to each other (with new ones to the right), *motion mode*, or M-mode, is obtained. The vertical axis will be depth in meters downwards, while the horizontal axis will be time in seconds pointing to the right. This mode can be useful when imaging heart valves, because the movement of the valves will make distinct patterns in the “image”. An example is shown in Figure 9.

6.4 B-mode

Brightness or B-mode is obtained by physically moving the scan line to a number of adjacent locations. The principle is shown in Figure 10. In this figure, the transducer is moved in steps mechanically across the medium to be imaged. Typically 100 to 300 steps are used, with a spacing between 0.25λ and 5λ. At each step, a short pulse is emitted followed by a period of passive registration of the echo. In order to prevent mixing the echoes from different scan lines, the registration period has to be

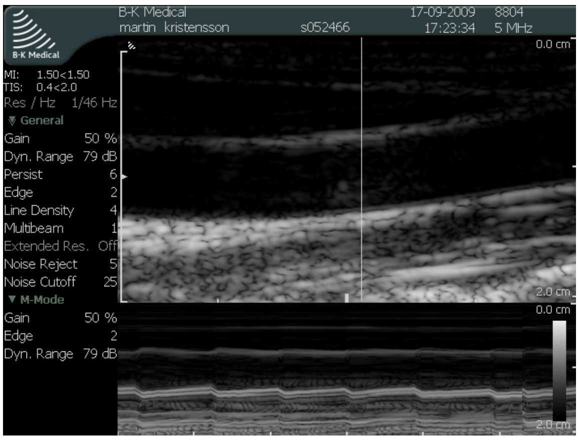

Figure 9 Screen dump of clinical ultrasound scanner used to image the carotid artery in the neck. Upper: the B-mode image. Lower: the M-mode image recorded along the vertical line in the B-mode image. Notice in the lower image, the change in location of the vessel walls due to the heart beat.

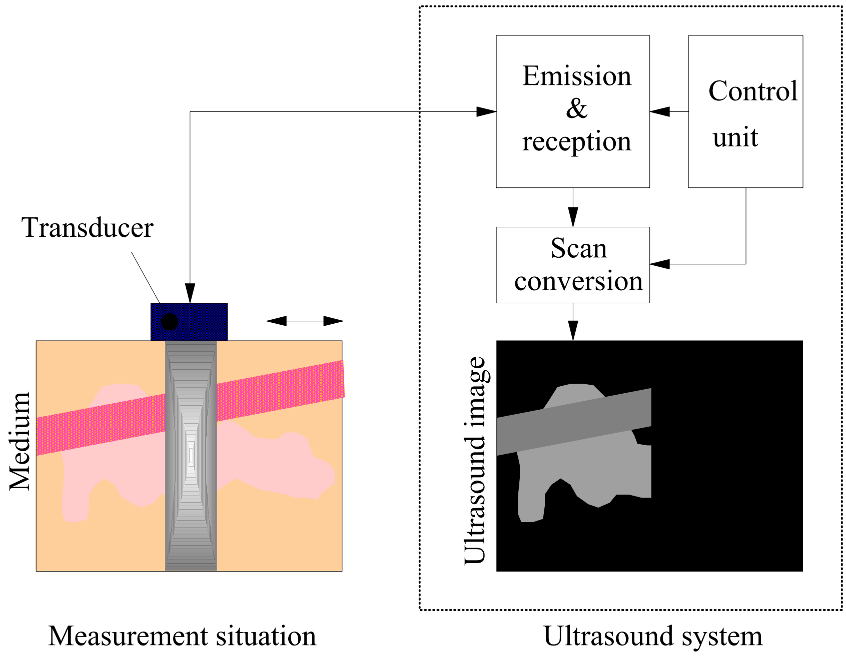

Figure 10 The principle of a simple B-mode ultrasound system. At this particular point in time, half of the image has been recorded.

long enough to allow all echoes from a given emitted pulse to be received. This will now be considered in detail.

Assume that the average attenuation of ultrasound in human soft tissue is α in units dB/MHz/cm. If the smallest echo that can be detected - on average - has a level of γ in dB, relative to the echo from tissue directly under the transducer, then the maximal depth from where an echo can be expected is $\gamma = \alpha f_0\, 2D_{max}$ or

$$D_{max} = \frac{\Upsilon}{2\alpha f_0} \tag{13}$$

Example: According to a rule of thumb, the average attenuation of ultrasound in human soft tissue is 1 dB/MHz/cm. Assume that $\gamma = 80$ dB. At $f_0 = 7.5$ MHz (13) gives $D_{max} = 5.3$ cm.

The time between two emissions will then be $T_r = 2D_{max}/c$, which is the time it take the emitted pulse to travel to D_{max} and back again. If there are N_l scan lines per image, then the frame-rate (number of images per second produced by the scanner) will be

$$f_r = (T_r N_l)^{-1}. \tag{14}$$

Example: For $N_l = 200$, $f_r = 70$ Hz a good deal more than needed to obtain “real-time” images (some 20 frames per second). However, an f_r of 70 Hz might not be an adequate temporal resolution, when studying heart valves. If the total image width is 40 mm, then the distance between adjacent scan lines is 40 mm / 200 = 0.2 mm. Please note that this number is not directly reflecting the spatial resolution size of the scanner, which is considered in Chapter 8.

Problem 7 If the frame rate is $f_r = 20$ Hz (a typical number for clinical use), how long time will be available for recording half an image as shown in Figure 10?

In order to better appreciate the dynamics of the recording situation, the live (MATLAB) version of Figure 11 shows the recording situation in extreme slow motion. It will be wise to consider this animation in detail. To help with this, a number of problems and quizzes are provided below:

Problem 8 Use a ruler (Danish: *lineal*) to check, if the green dots in Figure 11 are located correctly, when the red dot is at the location shown? And should the green dots decrease in amplitude as they travel back through tissue?

Problem 9 How much slower is the scanning performed in Figure 11, compared to normal clinical use?

Examples of clinical B-mode images can be seen in the chapter on clinical imaging in this Webbook.

7 Array transducers

The recording of a B-mode ultrasound image by *mechanical* movement of the transducer is now an old technique. Today most ultrasound systems apply array transducers, which consist of up to several hundreds of crystals, arranged along a straight or curved line. Se an example in Figure 11. The elements of the transducer array, or a subset of elements, are connected to a multichannel transmitter/receiver, operating with up to several hundred independent channels. The shape, direction and location of the ultrasound beam can then be controlled electronically (in the newest scanners completely by software) thereby completely eliminating mechanical components of the transducer. In the most

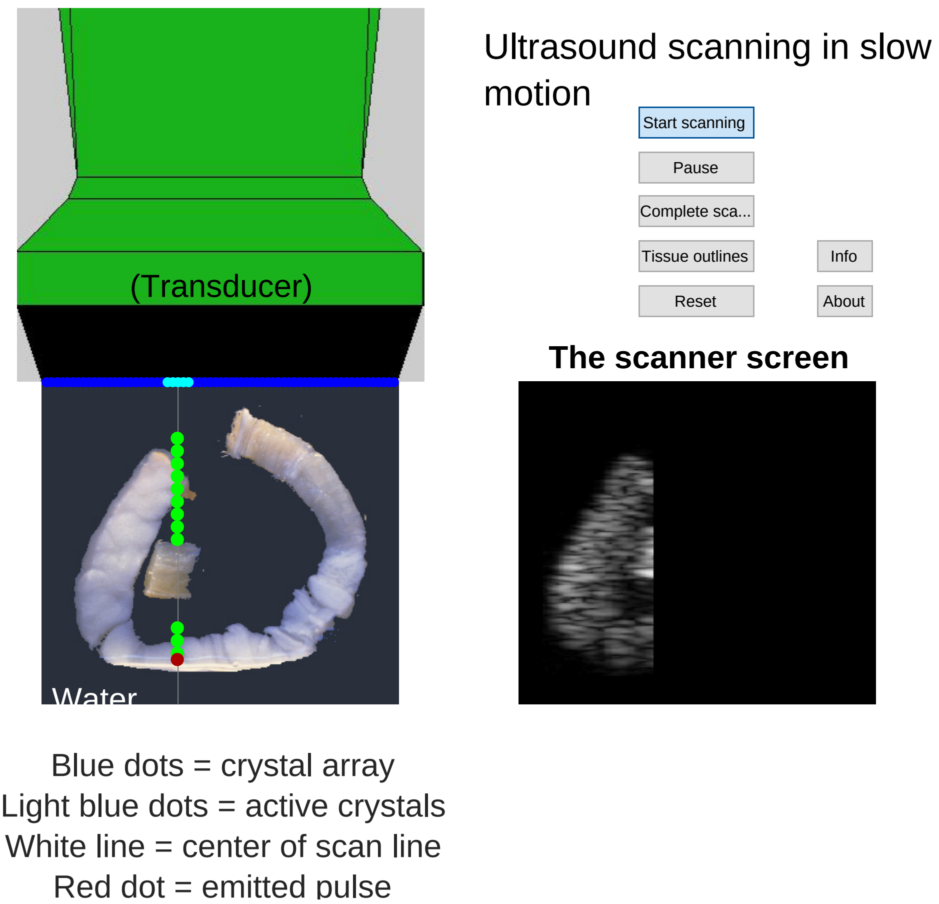

Figure 11 Illustration of the recording of a B-mode image. Left: The ultrasound transducer scanning a piece of animal tissue in water. The photograph of the tissue is made by later slicing the tissue and photographing the slice where the scanning took place. The red dot represents the emitted pulse, which decreases in amplitude the more tissue it penetrates. The green dots represents the echoes. Right: The screen of the scanner. The scan line is updated from left to right. Not all in this "drawing" is to scale.

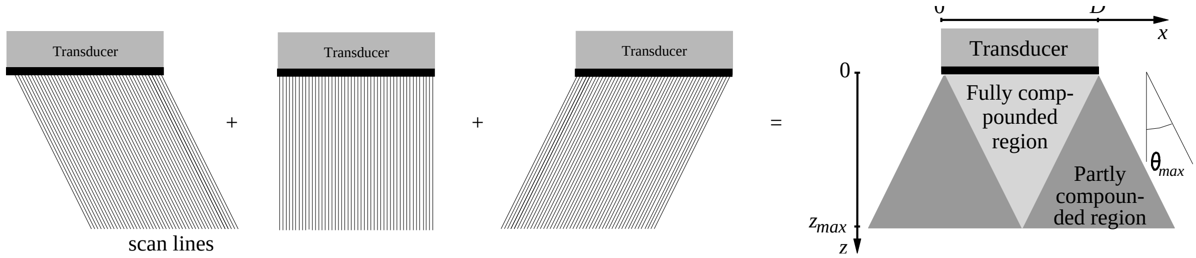

Figure 12 The principle of spatial compound imaging for $N_\theta = 3$. Three single-angle images are recorded from three different angles and then averaged to form the compound image. Inside the triangular region, the image is fully compounded; outside, it is less compounded.

flexible systems, the amplitude, waveform and delay of the pulses can be controlled individually and precisely.

Two different types of transducer systems exist: *Phase array systems*, where all elements are in use all the time. The beam is then steered in different directions to cover the image plane. In the *linear array systems*, a subset of elements is used for each scan line. From this subset a beam is created, and then translated by letting the subset of elements "scan" over the entire array. The latter can be observed (schematically) in Figure 11: The blue dots show all the crystals. The light blue dots show the active crystals, which are used for emitting a focused beam and receiving the echoes along the same beam.

8 Resolution size and point spread function

The resolution size of an imaging system can be assessed in many different ways. One way is to record an image of a small point target. The resulting image is called the *point spread function* (psf), *i.e.* an image which shows how much the image of a point target is "spread out", due to the limitations of the imaging system. The point target should preferably be much smaller than the true size of the psf. Another related way is to image two point targets with different separations, and see how close they can be positioned and still be distinguishable.

The –3 dB width of the psf in the vertical and horizontal image direction will then be a quantitative measure for the resolution size. The two directions correspond to the depth and lateral direction in the recording situation, respectively.

The resolution in the depth direction (axial resolution) can be appreciated from the echo signal in Figure 8. This echo signal was created by emitting a pulse with the smallest possible number of periods. The resolution size is equal to the length of the echo pulse from a point target, which in the present noted is assumed identical the emitted sound pulse. Thus, if the axis resolution size should be improved (decreased) the only possible way is to increase the centre frequency of the transducer. But increasing f_0 will increase attenuation as well, as discussed in Subsection 5.5. The consequence is that centre frequency and resolution size is always traded off.

The resolution size is treated in more detail in the chapter on image quality in this webbook.

9 Spatial compounding*

The array technique described in Subsection 7 can be used to implement so-called spatial compounding. In this technique, several images are recorded from different angles and then combined, to yield an image with some desirable properties, relative to the conventional B-mode image. The technique is illustrated in Figure 12. Because a single compound image consists of N_θ single-angle images, the frame-rate will be reduced by a factor of N_θ compared to B-mode imaging.

An example of a conventional B-mode image and the corresponding compound image is shown in Figure 13. If compared to the B-mode image, a number of (desirable) features become apparent:

The B-mode image has a quite “mottled” appearance, in the sense that the image consists of dots - roughly the size of the psf - on a black background. This is the result of the before mentioned constructive and destructive interference from closely spaced scatterers and reflectors, as illustrated in Figure 7. The phenomenon is commonly referred to as speckle noise. Speckle noise is a random phenomenon, and a given combination of constructive and destructive interference from a cloud of closely spaced scatterers is closely related to beam size, shape, orientation and direction. Thus, the interference pattern will change for the same tissue region when imaged from a different direction. If the change in view-angle is large enough, this interference patterns will be uncorrelated; so averaging of several uncorrelated single-angle images, will yield a reduction in speckle noise.

Because the ultrasonic echoes from interfaces vary in strength with the angle of incidence, the more scan angles used, the larger the probability that an ultrasound beam is perpendicular or nearly perpendicular to an interface, and the better the interface will be visualized.

The reduction in speckle noise and the improvement in visualization of interfaces give an image with a more smooth appearance, better contrast and better delineation of boundaries. This can be seen in Figure 13 (right).

Problem 10 In Figure 13 left, there are two bright dots at 9 o’clock and 10 o’clock, but only one at 10 o’clock in Figure 13 right. Why?

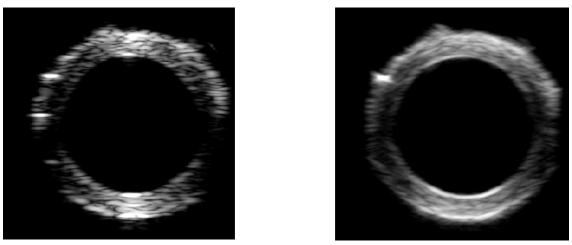

Figure 13 Left: Conventional image of a porcine artery. Right: Spatial compound image of the same porcine artery (average image of single-angle images from the angles: -21°, -14°, -7°, 0°, 7°, 14°, 21°).

10 Attenuation values

In Table 11, some attenuation values are given.

Table 11: Ultrasound attenuation of human tissues and other media at a frequency of 1 MHz.[6]

Material or tissue	Attenuation in dB/cm
Blood	0.18
Fat	0.6
Kidney	1.0
Muscle (across fibers)	3.3
Muscle (along fibers)	1.2
Brain	0.85
Liver	0.9
Lung	40.0
Skull	20.0
Lens	2.0
Aqueous humor	0.022
Vitreous humor	0.13
Water	0.0022
Castor oil	0.95
Lucite	2.0

11 Nomenclature

a Radius of transducer disk (m)

R Radius of curvature of spherically focused transducer (m)

λ Wavelength of ultrasound (m)

f_0 Centre frequency of emitted pulse (Hz)

c Propagation speed of ultrasound (m/s)

N_θ Number of single-angle images in spatial compound ultrasound

N_l Number of scan lines in an ultrasound image

$f_r = T_r^{-1}$ Pulse repetition frequency (Hz)

D_{max} Maximal depth (m)

$g_r(t)$ Received signal (V)

$|\tilde{g}_r(t)|$ Envelope of received signal (V)

α Attenuation (m^{-1})

p Pressure (Pa)

z Specific acoustic impedance (rayl = $kg/(m^2s)$)

ρ Physical density of medium (kg/m^3)

κ Compressibility of a medium (Pa^{-1})

12 Glossary

Refraction "The deviation of light in passing obliquely from one medium to another of different density. The deviation occurs at the surface of junction of the two media, which is known as the refracting surface. The ray before refraction is called incident ray; after refraction it is the refracted ray. The point of junction of the incident and the refracted ray is known as the point of incidence. [...]".[1]

Isotropic "Similar in all directions with respect to a property, as in a cubic crystal or a piece of glass."[1]

dB A magnitude variable, such as pressure, p, in Pa, can be written in as $20\log_{10}(p/p_{\text{ref}})$ dB, where p_{ref} is some given reference pressure, needed to render the argument to the logarithm dimensionless. Likewise intensities, I, can be written as: $10\log_{10}(I/I_{\text{ref}})$ dB.

13 Quizzes

Question 1

Consider two homogeneous media placed next to each other so that the interface is planar and large. In medium one, the density is 1000 kg/m3 and speed of sound is 1500 m/s. In medium two, the density is 1250 kg/m3 and the sound speed is 1200 m/s. What is the reflection coefficient between the two media?

Select the right answer:

A. This can only be answered, if the direction is specified.

B. 1.2 - 50.

C. 1.3 - 500.

Question 2

In the above question: If the ultrasound wave travels from medium 1 to 2, what is the pressure reflection coefficient?

Select the right answer:

A. 1.5 Mrayl.

B. -1.

C. 0.

D. 1

Question 3

If R is the pressure reflection coefficient and T is the pressure transmissions coefficient, what is correct?

Select the right answer:

A. R+T = 1.

B. T = Pt/Pi and R = Pr/Pi, where Pi = pressure of incident wave, Pr = pressure of reflected wave and Pt = pressure of transmitted wave.

Question 4

What happens to the red dot as it traverse along it's path?

Select the right answer:

A. There is no change

B. It grows in size

C. It shrinks in size

Question 5

Why does this change occur?

Select the right answer:

A. Because the material it passes through absorbs the signal.

B. Because the material it passes through attenuates the signal.

C. Because the specific acoustic impedance is larger than zero.

D. Because the specific acoustic impedance is zero.

E. Because the specific acoustic impedance is negative.

Question 6

Does water absorb just as much acoustic energy as oil?

Select the right answer:

A. Actually, water absorbs more of the acoustic energy than oil.

B. Actually, water absorbs the same amount of acoustic energy as oil.

C. Actually, water absorbs much less of the acoustic energy than oil.

Question 7

What does specific acoustic impedance signify?

Select the right answer:

A. In a homogeneous medium, it is defined as the ratio of pressure to particle velocity in a progressing plane wave

B. The higher the impedance, the higher the loss, just like in an electronic circuit

Question 8

In a previous quiz, the main theme was the diminishing size of the red dot, that represents the forward propagating pulse which experience attenuation throughout the medium. Should the green dots behave correspondingly?

Select the right answer:

A. Yes, they should diminish in size

B. No, they should stay the same size.

Question 9

As a rule of thumb, the attenuation is about 1 dB/cm/MHz. This would give an ultrasound image, which would be darker and darker as depth increases. What can be done to counteract this?

Select the right answer:

A. Display the image in dB

B. Use of Time-Gain-compensation (TGC). Here, by multiplying the received signal with the inverse attenuation function, the fading of the image as a function of depth can be counteracted (on average).

Question 10

Are there situations, where this automatic gain compensation gives "wrong" results?

Select the right answer:

A. Yes.

B. No.

Question 11

Which situations?

Select the right answer:

A. E.g., when scanning the (full) urine bladder.

B. E.g., when scanning the liver.

14 References

[1] Dorland's Illustrated Medical Dictionary. 27th edition. W. B. Saunders Co., Philadelphia, PA, USA. 1988.

[2] Kinsler LE, Frey AR, Coppens AB & Sanders JV: Fundamentals of acoustics. 3rd ed. John Wiley & sons, Inc. New York, NY, USA, 1982.

[3] Orofino, DP: Analysis of angle dependent spectral distortion in pulse-echo ultrasound. PhD dissertation, Department of Electrical Engineering, Worcester Polytechnic Institute, August 1992, USA.

[4] Kino, GS: Acoustic waves. Prentice-Hall, Inc. Englewood Cliffs, New Jersey, USA. 1987.

[5] Jensen, JA: Estimation of Blood Velocities Using Ultrasound. A Signal Processing Approach. Cambridge University Press, New York, 1996. ISBN 0-521-46484-6.

[6] https://wiki.engr.illinois.edu/download/attachments/44730411/table+1-11.jpeg?version=1

15 Solutions to selected problems

Problem 3: A possible way is to simulate the field with a given number of sources, and then see if the results change when the number of sources are increased (apart from scaling). If the number of sources can be doubled, or tripled (etc.) without a change in form, the number of sources are probably representative for the transducer surface.

Problem 4: Learning wise, it would be meaningless to provide an answer here. Instead, please write the resume yourself. Then wait two weeks, read the chapter again and compare with the resume you originally wrote.

Problem 5: –3 dB (or 3 dB lower).

Problem 6: The interface between z_1 and z_2 together with the interface between z_2 and z_3 generate an interference echo that is different in shape from the emitted signal (the slap of material denoted z_2 is thinner than the pulse length, thus the two echoes will always overlap in time).

Problem 7: 1/40 s.

Problem 8: Here you have to consider travel time and location of interfaces, in order to see if the green dots are placed correctly. Yes, the green dots should also decrease in amplitude as they pass through tissue.

Problem 9: Measure how much time it takes to finish one image (using the MATLAB animation). Calculate how much time it takes to record an image, when the frame rate is 20 Hz. Divide the two numbers.

Problem 10: The dot at 10'oclock that appears on both images is probably due to a micro vessel supplying blood to the arterial wall. The dot a 9'oclock that only appear on the single-angle image is likely to be a result of quite strong constructive interference.

16 Solution to quizzes

Q1: A, Q2: C, Q3: B, Q4: C, Q5: B, Q6: C, Q7: A, Q8: A, Q9: B, Q10: A, Q11: A.