Target Velocity Estimation with FM and PW Echo Ranging Doppler Systems — Part I: Signal Analysis

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***Abstract*—The theoretical foundation is presented for velocity estimation with a Doppler system, transmitting linear FM signals. The Doppler system possesses echo ranging capabilities and is evaluated in the context of Doppler ultrasound for blood velocity measurement. The FM excitation signal is formulated and the received signal is derived for a single moving particle. This signal is similar to the transmitted signal, but with modified parameters due to Doppler effect and range. The demodulated received signal is subsequently derived and analyzed. It is shown that, due to the Doppler effect, this is a linear sweep signal as well. Finally, the velocity and range information obtainable from one and two consecutively received signals are described, where the latter case establishes the basis for an FM Doppler system for blood velocity measurements.**

Nomenclature for Part I and Part II

B_a	Bandwidth of demodulated signal, FM-*fsm* system.
B_{FM}	Bandwidth of transmitted sweep signal.
d	Initial depth of acoustic particle.
$d^{(n)}$	Depth of acoustic particle at the onset of the nth transmission.
D	Nominal depth, i.e., depth of interest.
$D_{\max}$	Maximal depth from which detectable reflections are coming.
ΔD	Range resolution size along acoustic axis.
$\Delta D_{\min}$	Minimum axial resolution size.
Δd	Change in range.
$f_a^{(n)}$	Center frequency of *fsm* spectrum.
f_d	Doppler frequency.
f_w	Width of spectral window in FM-*fsm* Doppler system.
f_0	Center frequency of transducer.
f_1	Start frequency of transmitted sweep signal.
f_2	Stop frequency of transmitted sweep signal.
$\Delta f(v)$	Change in frequency as a function of velocity.
Δf_a	Shift between consecutive *fsm* spectra.
$\tilde{g}_m^{(n)}(t)$	Complex output signal from mixer.
$\tilde{g}_a^{(n)}(t)$	Complex output signal from demodulator.
$\tilde{g}_d(n)$	Complex Doppler signal, FM-*psm* system.
$g_r^{(n)}(t)$	Received signal from an acoustic particle.
$\tilde{g}_{\mathrm{ref}}(t)$	Complex reference signal.
$g_t(t), \tilde{g}_t(t)$	Transmitted signal and complex version of transmitted signal.
$\tilde{G}_a^{(n)}(f)$	Complex *fsm* spectrum, FM-*fsm* system
$\tilde{G}_{a,w}^{(n)}(f)$	Windowed complex *fsm* spectrum, FM-*fsm* system.
L	Number of transmissions.
M	Number of resolution cells.
n	Discrete time; transmission number.
S_d	Doppler sweep-rate.
S_r	Sweep-rate of received signal.
S_0	Sweep-rate of transmitted signal.
t	Local time.
t_m	Duration of transmitted signal.
t_s	Time from transmission to range gate (=$2D/c$).
T_{obs}	Observation time.
T_r	Sweep repetition time (= $2D_{\max}/c$).
v	Velocity of acoustic particle towards the transducer.
$\hat{v}$	Estimated velocity.
β	$\equiv (c+v)/(c-v)$.
γ	Frequency shift in cross correlation function.
$\hat{\gamma}_p$	Estimated frequency shift corresponding to maximum in cross covariance function.
$\Psi(t)$	Instantaneous frequency function (associated with signal with corresponding subscript).
τ	Global time.
$\theta(t)$	Time varying phasor (associated with signal with corresponding subscript).
φ	Constant phase.

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I. Introduction

In recent years, medical ultrasound scanners have been developed into highly sophisticated diagnostic equipment, based on the attractive noninvasive nature of ultrasound examinations and nonionizing properties of ultrasound. The high level of sophistication is particularly true for ultrasound Doppler equipment which typically must operate on the low-level noisy backscattered signals from blood, embedded in the much stronger reflections from the stationary structures. Consequently, any new approach for excitation signal generation and/or information extraction is potentially significant.

The early instrumentation for blood flow measurements was based on the *continuous wave (CW) Doppler* [1], [2] concept, where one transducer transmits a continuous tone and another receives the backscattered sound. The frequency of the backscattered signal from moving scatterers is Doppler shifted, and this shift can be detected by the receiver. As the transmitted signal is not coded (in time), no range information can be obtained from the received signal. To obtain velocity information about scatterers at selected ranges, amplitude modulation can be used. This is achieved with *pulsed wave (PW)* [1], [2] excitation, where short tone bursts are transmitted. The received signal from a given range can now be obtained by simple time discrimination.

With PW excitation, a series of coherent bursts are transmitted with fixed time intervals. Due to the Doppler effect, introduced by the motion of the scatterers, the received signal will be compressed or expanded. This has two effects: each individual burst at the carrier frequency will experience a Doppler compression or expansion, but the pulse repetition interval will also be Doppler shifted. The first effect will result in a slight velocity proportional frequency shift in the spectrum of the received signal relative to the transmitted spectrum. Due to the second effect, the characteristic features (the "signature") of consecutive received signal segments from the same range will undergo progressive time shifts with respect to the time of the transmission. This time shift corresponds to the continuing change in round trip travel time between transducer and scatterers during measurement.

Conventionally, the received signal is processed with a correlator and then sampled [1], [2]. Specifically, the cross correlation between each received signal and the fixed frequency source from which the coherent bursts are obtained is calculated and sampled to yield a *phase estimation*. The series of consecutive phase values forms the discrete cross correlation function which will oscillate with the Doppler frequency, where the spacing between the samples is considered to be the pulse repetition time. The larger the change in round trip travel time between consecutive received signals is the larger and is the change in consecutive phase values, and the higher is the frequency of the Doppler signal. Unfortunately, a phase shift larger than $\pm\pi$ cannot be distinguished unambiguously which is the reason for the well-known *velocity ambiguity* in correlation-based Doppler systems. This signal processing will be referred to as *phase shift measurement* (PW-*psm*), see Table I.

It is important to note that it is the time shift of the "signature" of the received signal that leads to the phase value, measured with the *psm* signal processing. This fact can easily be misunderstood since the velocity is determined from the Doppler frequency by using the well-known Doppler equation, which specifically relates velocity to the shift of the carrier frequency (for the individual bursts). However, the Doppler effect on the individual bursts is difficult to detect, but does in principle decrease the precision of the velocity estimate. Nonetheless, following convention, the word Doppler will be used for the systems described herein.

To avoid the velocity aliasing problem, another signal processing technique based on cross correlation between consecutive received signal segments has been proposed, [3], [5], [9], [11] whereby the previously described time shift is measured directly. From the time shift, the velocity can be estimated. This technique will be referred to as *time shift measurement* (PW-*tsm*), as shown in Table I. Since no phase interpretation is involved, no velocity ambiguity will exist.

TABLE I
OVERVIEW OF DOPPLER SYSTEMS FOR ASSESSMENT OF BLOOD VELOCITIES

Excitation signal	Trans-ducers	$\frac{P_{peak}}{P_{average}}$	Signal processing: Phase shift measurement	Signal processing: Cross correlation measurement
Pulsed wave[1], [2]	1	>> 1	PW-*psm*	PW-tsm
Random signal[17]	2	≅ 1	RS-*psm*	—
Pseudorandom noise[7], [8]	2	≅ 1	PRN-*psm*	—
Frequency modulation[19]—[22]	2	≅ 1	FM-*psm*	FM-*fsm*

PW excitation contains an inherent limitation of a completely different nature. This limitation concerns the relationship between *peak to average power ratio* and *minimum obtainable resolution size*. The minimum resolution size is determined by the bandwidth of the transmitted signal which in a PW system is linked to the time duration of the transmitted signal. In essence, a good range resolution requires a short pulse. On the other hand, if the average transmitted power is to be kept constant (i.e., maintaining a constant signal-to-noise ratio), reduced pulse duration will require higher transmitted peak power.

The implication of this in medical ultrasound can specifically be that the transmitted peak power must be limited in order for the equipment to comply with the regulations for ultrasound scanners. Furthermore, nonlinear effects definitely exist at intensities (spatial peak, temporal peak) above 10 W/cm^2 [6]. As the biological effect of low duty cycle ultrasound is still not fully known, the smallest possible peak power is always desirable. *In vitro* experiments with a commercial scanner have recently showed that cavitation can arise in hydrophobic polystyrene spheres at intensities used with pulse-echo diagnostic ultrasound equipment [14].

Only for fixed frequency tone bursts is there a clearly established relationship between time duration of the transmitted signal and the bandwidth. To remove this connection between time duration and bandwidth, coded signals can be used. Several systems utilizing coded signals (as in radar) have been proposed. Examples of such coded signals are: *random signal* [17] and *pseudorandom noise* [7], [8] as shown in Table I. The general disadvantages of these excitation signals are: the need for two transducers, the so-called range sidelobes (described in the companion paper [22]) and the risk of velocity aliasing, as they in general are used in connection with the *psm* method.

As a new approach to target velocity measurements, this paper will investigate the use of excitation signals in the form of repetitive linear sweeps. Among the specific advantages are the possibility for eliminating the aliasing problem and a duty cycle which approaches unity. Such sweep signals belong to the group of coded excitation signals with long time duration and will thus feature lower peak to average

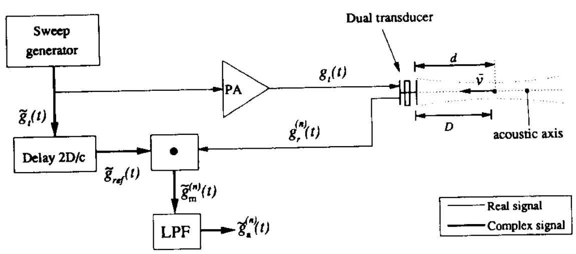


Fig. 1. Concept of the FM Doppler system. A sweep generator creates a complex repetitive coherent linear sweep signal. This signal is transmitted with one transducer while a second transducer receives backscattered signal. The received signal is multiplied with a time delayed complex replica of the transmitted signal and low-pass filtered. The velocity information is extracted from this signal. A single particle is travelling towards the transducer with constant velocity $\bar{v}$. PA is a power amplifier.

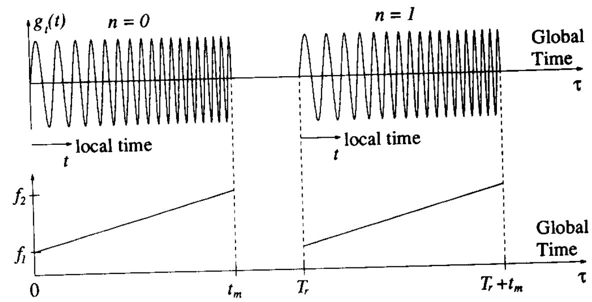


Fig. 2. Upper graph: the first and second transmitted time signals as they will be used in the FM Doppler. Lower graph: the corresponding instantaneous frequency. The local and global time axes are indicated.

power ratio, but also range sidelobes. FM excitation signals in Doppler ultrasound have to the best of our knowledge only been proposed [12], [15], [16], but never reported for use in measuring blood velocity profiles [1], and the feasibility of employing such signals has been disputed [1, p. 46]. It will be demonstrated in the companion paper [22], however, that two types of signal processing methods, closely related to the processing techniques in PW Doppler, are possible in connection with FM excitation, as listed in Table I. It is also noted that a B-scan imaging system has been constructed, based on FM chirps [13].

When the *psm* method is used (FM-*psm*) the velocity aliasing still exists. However, it will be shown that a method corresponding to the *tsm* method exists in the frequency domain, thus eliminating the velocity aliasing. As a frequency shift will be measured with this technique, it will be referred to as: *frequency shift measurement* (FM-*fsm*). Another potential advantage of using a linear FM sweep is the possibility for easy precompensation for the transducer transfer function and, to some degree, for frequency dependent attenuation and scattering from blood.

II. Concept of FM Doppler System

The basic concept of the FM Doppler system is shown in Fig. 1. The sweep generator creates a repetitive linear sweep signal. The signal is generated in quadrature, and the real part of the signal is amplified and applied to the transmitting transducer. The backscattered signal from a single scatterer, moving along the acoustic axis with constant velocity $\overline{v}$ and initial position d at $t = 0$, is acquired with a separate receiving transducer.

The received signal is multiplied (in quadrature) with a delayed (or equivalently, frequency shifted) version of the transmitted signal where the delay corresponds to the round trip travel time to the depth of interest, D. The low-pass filtered output, $\tilde{g}_a^{(n)}(t)$ contains the velocity information for scatterers located in the vicinity of D. $\tilde{g}_a^{(n)}(t)$ may be processed in two distinctly different ways, leading to two types of FM Doppler systems.

In order to remove stationary echo components, a stationary echo canceller must be applied. This is not indicated in the general layout in Fig. 1 because the location of this device is different in the two FM Doppler systems. However, incorporating a stationary echo canceller on the RF signals requires that the transmitted signals be coherent, meaning that the phase behavior of consecutive sweeps are identical.

The next three sections will describe the transmitted signal, the received signal, and the difference frequency component of the mixer output signal (demodulated signal).

III. Transmitted Signal

The basis of the FM Doppler system is transmission of repetitive coherent swept frequency signals as depicted in Fig. 2. The signals are transmitted according to a sweep repetition time, T_r, analogous to the pulse repetition time in a PW system. The duration of the sweep signals, t_m, is much longer than the bursts used with PW excitation. To avoid mathematical intractability in the derivations to follow, *the signals are all described in terms of local time*, t, relative to the onset of the corresponding transmission. The general form of the transmitted signal is

$$g_t(t) = A\sin(2\pi f_1 + \pi S_0 t^2) \quad 0 \le t \le +t_m \quad n \in [0; L-1] \tag{1}$$

where f_1 is the start frequency, S_0 is the sweep rate, and the integer n denotes the nth transmission out of a total of L transmissions. n-notation is left out in the transmitted signal as it is identical for all transmissions. For a given n, $g_t(t)$ is a *linear frequency modulated tone* of amplitude A, also called a *sweep signal* or a *chirp*. It is noted that if in (1), $S_0 = 0$ and t_m is small relative to T_r, $g_t(t)$ defaults to PW excitation. If the instantaneous frequency at $t = t_m$ is f_2 the sweep rate is $S_0 = (f_2 - f_1)/t_m$. The bandwidth of such an FM signal will roughly be $B_{\text{FM}} = f_2 - f_1$. The instantaneous frequency in hertz, $\Psi_t(t)$, of the analytic version of the transmitted signal, $\tilde{g}_t(t)$, increases linearly with time and is given as

$$\Psi_t(t) = f_1 + \left(\frac{f_2 - f_1}{t_m}\right)t = f_1 + S_0 t. \tag{2}$$

In the following, if the term instantaneous frequency for a real signal is stated, the instantaneous frequency of the corresponding analytical signal is implied. We shall also assume an ideal ultrasound transducer in all aspects: no frequency dependent sensitivity; linear conversion from electric energy to acoustic energy and vice versa. The medium is assumed perfectly

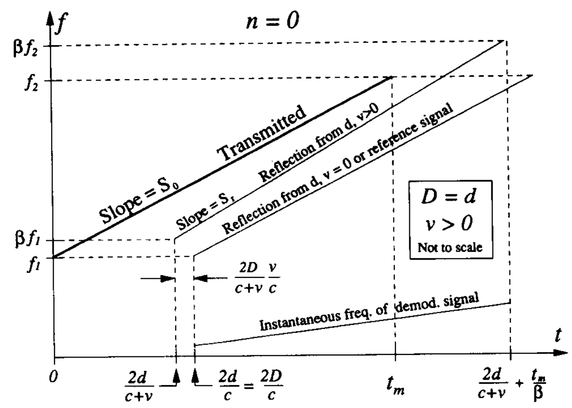


Fig. 3. Detailed illustration of the depth dependent demodulation concept for one transmission/reception. The instantaneous frequency of four signals are shown: the transmitted signal, the signal from a stationary particle at the depth d, the signal from a moving particle at the depth d and the demodulated signal $\tilde{g}_a^{(0)}(t)$. The terms on this figure are not approximated.

elastic so that the pressure wavefront throughout the medium is proportional to $g_t(t)$, apart from propagation delays. At the depth of interest, the wavefront is further assumed perfectly plane. Other frequency dependent factors are ignored, such as the radiation characteristics and diffraction effects of physical transducers, phase cancellations effects, frequency dependent attenuation in the medium, and frequency dependent scattering from blood.

IV. Received Signal from One Moving Particle

Consider one acoustic particle at the depth d at $t = 0$, moving with constant velocity $\overline{v}$ along the acoustic axis towards the transducer, in a medium where the speed of sound is c, as illustrated in Fig. 1. The received signal from this particle, given the transmission signal in (1), will now be derived. Considering $g_t(t)$ for $n = 0$, we observe that the instantaneous frequency of the transmitted signal is f_1 at $t = 0$ and f_2 at $t = t_m$. For the corresponding received signal from the particle, the instantaneous frequency, $\Psi_r(t)$, at the beginning and end points (see Fig. 3) of the received signal, respectively, will be [19]:

$$\Psi_r\left(\frac{2d}{c+\nu}\right) = \beta f_1 \qquad \Psi_r\left(\frac{2d}{c+\nu} + \frac{t_m}{\beta}\right) = \beta f_2 \tag{3}$$

where $\beta = (c + v)/(c - v)$ is the Doppler expansion/compression factor. The two frequencies in (3) correspond to the start and stop frequencies of the original transmitted signal plus the corresponding Doppler shift at this frequency. This is depicted in Fig. 3. The received signal thus has a sweep rate, S_r, equal to

$$S_r = \frac{\beta f_2 - \beta f_1}{\left(\frac{2d}{c+v} + \frac{t_m}{\beta}\right) - \left(\frac{2d}{c+v}\right)} = \beta^2 S_0. \tag{4}$$

When $v << c$, (4) can be linearized so that $S_r \cong (1+4v/c)S_0$.

Alternatively, the complete expression for the received signal can be obtained by introducing the expression for the time dependent round trip travel time [19]:

$$t_t(t) = \beta\left(t - \frac{2d}{c+v}\right) \tag{5}$$

into (1) and using: $d^{(n)} = d - nvT_r$, where $d^{(n)}$ indicates the location of the scatterer at the onset of the nth transmission. Assuming that the backscattering coefficient of the acoustic particle is r, the received signal is

$$\begin{aligned} g_r^{(n)}(t) &= r\,A\,g_t\left(\beta\left(t - \frac{2d^{(n)}}{c+v}\right)\right) \\ &= r\,A \sin\left(2\pi\left(\beta f_1 - \frac{2d^{(n)}\beta}{c-v}S_0\right)t + \pi S_r t^2 + \varphi_r^{(n)}\right) \end{aligned} \tag{6}$$

where

$$\frac{2d^{(n)}}{c+v} \le t \le \frac{2d^{(n)}}{c+v} + \frac{t_m}{\beta},$$

$$\varphi_r^{(n)} = -2\pi f_1 \frac{2d^{(n)}}{c-v} + \pi S_0\left(\frac{2d^{(n)}}{c-v}\right)^2 \tag{7}$$

and $n \in [0; L-1]$. Equation (6) is the general equation for the received signal from one particle, given a repetitive chirp excitation. Note that this equation satisfies (4) in terms of having the correct sweep rate as expected. If $v = 0$, (6) yields a time delayed version of the transmitted signal. This time delay can be interpreted as a frequency shift, which is the basis for time delay spectrometry [18]. Also note that for $S_0 = 0$ and t_m is appropriately short, (6) indicates the received signal given PW excitation. If further $L = 1$ and $t_m \to \infty$, then (6) corresponds to the received signal from CW excitation.

V. Analysis of Demodulated Signal

Before the two approaches to processing the FM Doppler signals are introduced, a complete analysis of the mixer output signal, $\tilde{g}_m^{(n)}(t)$ in Fig. 1 will be presented. The sum frequencies of $\tilde{g}_m^{(n)}(t)$ are assumed to be filtered out by an ideal low-pass filter, leaving only the signal component with the difference frequencies, $\tilde{g}_a^{(n)}(t)$, for this analysis. It should be noted that in the description of the two FM Doppler systems [22] the low-pass filtering is shown only implicitly in the form of an integration (FM-*psm*) or a spectral windowing (FM-*fsm*).

The reference signal, $\tilde{g}_{\mathrm{ref}}(t)$, is a unity amplitude, analytic version of the transmitted signal in (1), delayed by $t_s = 2D/c$ seconds, and is given in (8)

$$\begin{aligned} \tilde{g}_{\mathrm{ref}}(t) &= \tilde{g}_t(t - t_s) \\ &= j\ \exp\{-j(2\pi f_1(t - t_s) + \pi S_0(t - t_s)^2)\} \\ &= j\ \exp\{-j(2\pi(f_1 - S_0 t_s)t + \pi S_0 t^2 + \varphi_{\mathrm{ref}})\} \\ &\quad t \ge t_s \end{aligned} \tag{8}$$

where $\varphi_{\mathrm{ref}} = \pi S_0 t_s^2 - 2\pi f_1 t_s$ and n notation is ignored. The real part of this signal (apart from the amplitude and the sign) is what would be received from a stationary, perfect reflector at depth D in an ideal medium. The instantaneous frequency of such a signal is also indicated in Fig. 3. The exponential in (8) is multiplied with a "j" (corresponding to the addition of the phase term $+\pi/2$). With this form, a scatterer velocity towards the transducer will produce a positive Doppler shift as well as a positive shift in the instantaneous frequency of the demodulated signal. Multiplying the complex reference signal with each received signal and low-pass filtering with a gain of 2 yields for the nth demodulated signal

$$\tilde{g}_a^{(n)}(t) = 2LF\{\tilde{g}_{\text{ref}}(t)g_r^{(n)}(t)\}. \tag{9}$$

In order to simplify the derivation of an explicit expression for $\tilde{g}_a^{(n)}(t)$, the phase of the reference and received signals will be used. The time varying phase of the reference signal, $\tilde{g}_{\text{ref}}(t)$, given in (8) is

$$\theta_{\text{ref}}(t) = 2\pi(f_1 - S_0T_s)t + \pi S_0 t^2 + \varphi_{\text{ref}}. \tag{10}$$

The time varying phase of the nth received signal, with $d^{(n)} = d - vnT_r$ inserted, is

$$\theta_r^{(n)}(t) = 2\pi\left(\beta f_1 - \frac{2(d - vnT_r)\beta}{c - v}S_0\right)t + \pi S_r t^2 + \varphi_r^{(n)}. \tag{11}$$

Assuming an ideal low-pass filter, (10) and (11) can be used to rewrite (9) to obtain $\tilde{g}_a^{(n)}(t)$ in an explicit form

$$\begin{aligned}\tilde{g}_a^{(n)}(t) &= 2LP\left\{je^{-j\theta_{\text{ref}}}r\,A\sin(\theta_r^{(n)})\right\}\\ &\cong r\,A\ \exp[j(\theta_r^{(n)} - \theta_{\text{ref}})]\\ &= r\,A\exp\left\{j\left(2\pi\left((\beta - 1)f_1\right.\right.\right.\\ &\quad \left.+\left(t_s - \frac{2(d - vnT_r)\beta}{c - v}\right)S_0\right)t\\ &\quad \left.\left.+\pi(\beta^2 - 1)S_0t^2 + \varphi_a^{(n)}\right)\right\}\end{aligned} \tag{12}$$

where $\varphi_a^{(n)} = \varphi_r^{(n)} - \varphi_{\text{ref}}$ accommodate all the constant phase terms. This signal, which is a chirp, will be analyzed next. The signal exists inside the following approximate range

$$t_s < t < t_s + t_m \tag{13}$$

where the exact range for t when $n = 0$ can be observed in Fig. 3. The instantaneous frequency in hertz of the demodulated signal in (12), $\Psi_a^{(n)}(t)$, can be found to be

$$\begin{aligned}\Psi_a^{(n)}(t) &= \frac{2v}{c - v}f_1 + \frac{2d}{c}S_0\\ &\quad + 2(vnT_r - d)\frac{c + v}{(c - v)^2}S_0 + \frac{4vc}{(c - v)^2}S_0 t\end{aligned} \tag{14}$$

where $\Psi_a^{(n)}(t)$ is valid only in the approximate time interval given in (13). By shifting $\Psi_a^{(n)}(t)$ in time by t_s a new function $\Psi_a^{(n)}(t + t_s)$ is obtained which exists in the interval $[0; t_m]$. If this function is written so that the contribution of $(D - d)$ to the instantaneous frequency appears explicitly, one obtains

$$\begin{aligned}\Psi_a^{(n)}(t + t_s) &= \frac{2v}{c - v}f_1 + 2D\frac{v}{c}\frac{c + v}{(c - v)^2}S_0\\ &\quad + 2(D - d)\frac{c + v}{(c - v)^2}S_0 + 2vnT_r\frac{c + v}{(c - v)^2}S_0\\ &\quad + \frac{4vc}{(c - v)^2}S_0 t.\end{aligned} \tag{15}$$

It is seen from (15) that the demodulated signal is also a linear sweep signal with an initial frequency (the first four terms) and a sweep rate (the fifth term). The five terms will now be analyzed.

The **first** term is simply the Doppler shift corresponding to the start frequency, f_1, of the transmitted signal.

The **third** term is the ***Doppler shifted* frequency offset** arising from the particle being $|D - d|$ away from the nominal depth D. This term is thus the position term relative to the nominal depth. When $d = D$, i.e., the particle is at D at $t = 0$, this term vanishes.

By setting $d = D$ and noting the above observations for the third term, we see that the **second** term may now be interpreted as a ***Doppler shifted* frequency offset** due to the fact that the reference signal is delayed $2D/c$ and not $2D/(c + v)$. The result is that the start-points of the reference and received signals from the nominal depth do not coincide. This detail is depicted in Fig. 3.

The **fourth** term represents the introduced offset, as the initial position of the particle has changed from the first to the nth measurement. This term is also Doppler shifted.

The **fifth** term represents a sweep component which is proportional to v and can thus be interpreted as the incurred Doppler shift of the sweep rate, in contrast to a Doppler frequency shift. This term will be called the ***Doppler sweep rate***:

$$S_d = \frac{4vc}{(c - v)^2}S_0 \cong \frac{4v}{c}S_0. \tag{16}$$

The approximation applies when $v << c$. Note that the polarity of terms 1, 2, 4, and 5 depends on v, while the polarity of term 3 depends on the initial position relative to the nominal depth. For $v = 0$, (15) yields a constant frequency as expected, and in the special case when $d = D$, this constant will be zero (i.e., $\tilde{g}_a^{(n)}(t)$ is a dc signal). When $S_0 = 0$, (15) yields the same (instantaneous) frequency as the Doppler frequency encountered in CW and PW systems.

The complexity of (15) results from the fact that $\tilde{g}_a^{(n)}(t)$ contains the difference in instantaneous frequency of two chirps with different start frequencies, sweep rates, start times, and durations.

Making the approximation $v << c$, (15) simplifies to

$$\begin{aligned}\Psi_a^{(n)}(t + t_s) &\equiv \frac{2v}{c}f_1 + 2D\frac{v}{c^2}S_0 + 2\frac{D - d}{c}S_0\\ &\quad + 2\frac{vnT_r}{c}S_0 + S_d t.\end{aligned} \tag{17}$$

For a linear sweep with an envelope that is symmetric with respect to frequency, the spectrum will be centered around the frequency that corresponds to the instantaneous frequency at the middle of the envelope. Applying this to (17) yields a center frequency $f_a^{(n)}$:

$$\begin{aligned}f_a^{(n)} &\cong \Psi_a^{(n)}\left(t_s + \frac{t_m}{2}\right) \cong \frac{2v}{c}f_1 + 2D\frac{c}{c^2}S_0\\ &\quad + 2\frac{D - d}{c}S_0 + 2\frac{vnT_r}{c}S_0 + S_d\frac{t_m}{2}\end{aligned} \tag{18}$$

which is dependent on range, and, to a lesser degree, on velocity.

To illustrate the magnitude of the individual terms in (17), typical values for the variables will now be given. With $c = 1500$ m/s, consider the following values: $f_1 = 3.5$ MHz, $f_2 = 7.5$ MHz, $D_{\max} = 0.2$ m, $D = 0.1$ m, $T_r = t_m + 2D_{\max}/c = t_m + 267\mu$s, and the following parameters for the particle: $v = 1$ m/s, and $d = 0.1005$ m. If $t_m = 100\mu$s, then $S_0 = 4 \cdot 10^{10}$ Hz/s. The first four terms in (17) are then 4.7, 3.6, −26.7, and $9.8n$ kHz, respectively. The last term sweeps from 0 to 10.6 kHz over t_m. It can be concluded that none of the above terms are negligible nor dominate to a high degree. We are now going to examine further how range and velocity are encoded in the demodulated signal.

*A. Information from Demodulated Signal Due to **One** Transmission*

As seen from (17), both range and velocity information is encoded into the instantaneous frequency of the received (and demodulated) signal. It is instructive to investigate under which circumstances the Doppler information can be extracted from the instantaneous frequency, in order to formulate appropriate signal processing schemes usable in FM Doppler systems. This may be done by looking at: i) the initial instantaneous frequency; and ii) the instantaneous frequency as a function of time. For a single received signal ($n = 0$), the initial frequency is

$$\Psi_a^{(0)}(t_s) \cong \frac{2v}{c} f_1 + 2D\frac{v}{c^2}S_0 + 2\frac{D-d}{c}S_0. \quad (19)$$

Since (19) contains two unknowns, v and d, neither can be determined from one measurement. However, for velocities and distances in the physiological range ($0 < v < 2$ m/s and $0 < D < 0.1$ m), the third term dominates, and (19) will, therefore, directly indicate the range of the particle. Nonetheless, the corruption of depth information by these smaller velocity terms is inevitable and is an unavoidable consequence of using FM signals. The implication of the presence of these terms will be described later for both the FM-*psm* and the FM-*fsm* Doppler systems. $\Psi_a^{(0)}$ versus time is obtained by including the Doppler sweep rate:

$$\Psi_a^{(0)}(t_s + t) \cong \frac{2v}{c} f_1 + 2D\frac{v}{c^2}S_0 + 2\frac{D-d}{c}S_0 + S_d t. \quad (20)$$

In (20), range is translated into frequency — and for the last term — velocity is translated into sweep rate. Note that the concept of using frequency for range discrimination is intrinsic in swept frequency measurement systems, such as time delay spectrometry [18]. In pulse measurements the contrasting technique is time discrimination. It can be observed from (20) that for $v = 0$, only the third term remains, which is time independent and contains the range information. Thus if the effect of velocity is considered negligible, a bandpass filter can be applied to the signal in (12) to obtain (20), such that the signal from scatterers at a given range, specified by $|D - d|$, is extracted.

The extraction of velocity information from (20) will be considered next. The velocity information is encoded in two ways: i) as change in the initial frequency (the first two terms in (20)), and ii) as rate of change in frequency (the last term in (20)). As discussed, the change in initial frequency due to velocity is masked by the range information, and the velocity, therefore, cannot be found from the initial frequency. However, the Doppler sweep rate, S_d, is a linear function of velocity only (in a nondispersive medium).

It can thus be observed that the Doppler effect, using linear FM excitation, is manifested in a sweep rate, S_d, of the demodulated signal in the same way that the Doppler effect, using CW excitation, is manifested in a frequency shift. The important question, however, is whether the velocity can be estimated reliably from the Doppler sweep rate. Considering only one moving particle, a measure of the spectral extent of the demodulated signal is $S_d t_m$. This quantity must be much larger than the spectral "smearing" due to the finite duration of the signal which is $1/t_m$. Thus:

$$S_d t_m >> \frac{1}{t_m} \Rightarrow S_d t_m^2 >> 1 \Rightarrow v >> \frac{c}{4(f_2 - f_1)t_m}. \quad (21)$$

Applying the parameters used in the numerical example, (21) reveals that the conditions are met for $v >> 0.94$m/s, i.e., reliable velocity estimation can in general only be obtained for velocities well above typically occurring physiological blood velocities [20]. Although it may be possible to increase t_m and thereby lower the limit of the useful velocity range, this is not feasible to the extent that a clinically useful system can be based on information from the received signal due to one transmission. Note, however, that no aliasing takes place in this technique for velocity estimation and that within the limitations in (21), the velocity can be obtained from one measurement. If a stationary echo canceller (SEC) is employed, two transmissions will be needed, but it can be shown [19] that the same limitations, as discussed previously, will apply. To obtain a linear relationship between velocity and a parameter extracted from the received signal, a different approach must be considered.

*B. Information from Demodulated Signal Due to **Two** Transmissions*

From (17) it can be seen that only the fourth term changes from one sweep to the next. Therefore, by determining the difference between *the center frequency* for the nth demodulated signal, $f_a^{(n)}$, and the center frequency for the $(n+1)$th demodulated signal, $f_a^{(n+1)}$, we have

$$\Delta f_a = f_a^{(n+1)} - f_a^{(n)} \cong 2\frac{vT_r}{c}S_0. \quad (22)$$

An identical result is obtained when the difference between the *instantaneous frequencies* of the nth and the $(n+1)$th sweeps is determined. Equation (22) is a key result which opens the possibilities for an FM Doppler system, based on FM-*psm*) or on frequency shift measurements (FM-*fsm*). These systems are presented in Part II [22] of this paper. The two approaches are analogous to PW Doppler systems, PW-*psm* and PW-*tsm*, respectively, apart from the *time to frequency conversion factor*, S_0, for the FM-*fsm* Doppler system. Δf_a may be obtained in different ways, one of which is spectral cross correlation.

VI. Conclusions

This paper has presented the basic foundation for target velocity measurement by means of repetitive linear coherent FM excitation signals. The received signal from a single particle, given this excitation, was derived and analyzed. This signal is a sweep, with a *start frequency, start phase, and sweep rate* that is dictated by the velocity of the particle (Doppler effect) and distance from the transducer. The demodulated signal was analyzed as well, and it was found that the center frequency of the spectra of two consecutive signals was shifted in frequency by an amount dictated only by the velocity of the particle. This key result will be utilized in Part II [22] of this paper to formulate two FM Doppler systems.

In a realistic medical measurement situation, the frequency dependent attenuation of the intervening tissue medium and the frequency dependent backscattering of blood will produce an amplitude modulation on the received signal, $g_r(t)$, as will the transducer's own frequency response. These effects can to some extent be compensated for by varying the amplitude of the excitation signal to the ultrasound transducer during the sweep. Regarding the information obtained from the demodulated signal due to *one transmission*, these frequency dependent factors will likely give a small reduction in the accuracy of the extracted information. However, with respect to the information obtained from the demodulated signal due to *two transmissions*, the frequency dependent factors are expected to give only a very minor degradation in performance, as these factors are common for all received signals.

References

[1] P. Atkinson and J. P. Woodcock, Doppler Ultrasound And Its Use In Clinical Measurements. London, U.K.: Academic, 1982.

[2] D. W. Baker, F. K Forster and R. E. Daigle, "Doppler principles and techniques," in *In Ultrasound: Its Applications in Medicine and Biology*. Amsterdam, The Netherlands: Elsevier Scientific, ch. III, pp. 161–287, 1978.

[3] M. Bassini *et al.*, "An ultrasonic non-invasive blood flowmeter based on cross-correlation techniques," in *Proc. Ultrasonics Int. Conf. and Exhibition*, Graz, Austria, May 1979, pp. 273–276.

[4] B. Boashash, "Estimation and interpreting the instantaneous frequency of a signal - Part I: Fundamentals," *Proc. IEEE*, vol. 80, pp. 520–538, Apr. 1992.

[5] O. Bonnefous and P. Pesqué, "Time domain formulation of pulse Doppler ultrasound and blood velocity estimation by cross-correlation," *Ultrason. Imaging*, vol. 8, no. 1, pp. 73–85, Jan 1986.

[6] E. L. Carstensen and T. G. Muir, "The role of nonlinear acoustics in biomedical ultrasound," in *Tissue Characterization with Ultrasound. Volume I, Methods*, J. F. Greenleaf, ed. Boca Raton, FL: CRC, 1986, pp. 57–79.

[7] D. J. Cathignol, "Signal-to-clutter ratio in pseudo random Doppler flowmeter," *Ultrason. Imaging*, vol. 8, no. 4, pp. 272–284, Oct. 1986.

[8] D. J. Cathignol, C. Fourcade and J-Y. Chapelon, "Transcutaneous blood flow measurements using pseudorandom noise Doppler system," *IEEE Trans. Biomed. Eng.* vol. BME–27, pp. 30–36, Jan. 1980.

[9] D. Dotti *et al*, "Blood flow measurements by ultrasound correlation techniques," *Energia Nucleare*, vol. 23, no. 11, pp. 571–575, Nov. 1976.

[10] R. J. Doviak and D. S. Zrnić , "The Doppler weather radar," in *Aspects of Modern Radar*, Eli Brookner, Ed. Norwood, MA: Artech House, 1988.

[11] S. G. Foster, P. M. Embree, and W. D. O'Brien, "Flow velocity profile via time-domain correlation: error analysis and computer simulation," *IEEE Trans. Ultrason., Ferroelec., Freq. Contr.*, vol. 37, pp. 164–175, May 1990.

[12] D. Franklin, S. A. Borders, and W. S. Kemper, "Frequency modulated, pulsed, range gated sonar method for soft tissue imaging and blood flow measurement," in *Proc. 1st Triennial Meeting of the WFUMB and AIUM*, 1977, pp. 1831–1833.

[13] R. C. Heyser *et al.*, "Medical ultrasound imager based on time delay spectrometry," *Ultrason.*, vol. 27, pp. 31–38, Jan. 1989.

[14] C. K. Holland *et al.*: "In vitro detection of cavitation induced by a diagnostic ultrasound system," *IEEE Trans. Ultrason., Ferroelec., Freq. Contr.*, vol. 39, pp. 95–101, Jan. 1992.

[15] W. S. Kemper, D. Franklin, and S. F. Vatner, "Practical implementation of the FM ultrasonic flowmeter," in *Proc. 24th ACEMB-Int.* Las Vegas, NV, Oct. 31–Nov. 4, 1971, p. 269.

[16] K. McCarty and J. P. Woodcock, "Frequency modulated ultrasonic Doppler Flowmeter." *Med. Bio. Eng.* pp. 59–64, Jan. 1975.

[17] V. L. Newhouse and P. J. Bendick, "Analysis of random signal blood flow measurement," in *Proc. IEEE Ultrasonics Symp. Proc.*, 1973, pp. 94–97.

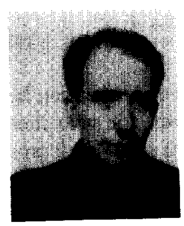
[18] P. C. Pedersen, P. A. Lewin, and L. Bjorno, "Application of time delay spectrometry for calibration of ultrasonic transducers," *IEEE Trans. Ultrason., Ferroelec., Freq. Contr.*, vol. 35, pp. 185–205, Mar. 1988.

[19] J. E. Wilhjelm, "Ultrasound FM Doppler system for flow profiling with comparison to PW Doppler systems." Ph.D. dissertation, Dept. of Biomed. Eng., Worcester Polytechnic Institute, MA, 1991.

[20] J. E. Wilhjelm and P. C. Pedersen, "Coherent FM Doppler system," in *Proc. IEEE UFFC Ultrasonic Symp.*, Oct. 1989, pp. 903–906.

[21] J. E. Wilhjelm and P. C. Pedersen, "FM Doppler systems - Experimental results," in *Proc. IEEE UFFC Ultrasonics Symp.*, Dec. 1990, pp. 1553–1556.

[22] J. E. Wilhjelm and P. C. Pedersen, "Target Velocity Estimation with FM and PW Echo-Ranging Doppler Systems, Part II - Systems analysis." pp. 373-380, this issue.



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