

# **31545 Medical Imaging systems**

## Lecture 9: Simulation of ultrasound systems and non-linear imaging

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1

## **Simulation of ultrasound systems and non-linear imaging**

1. Discussion assignment from last lecture
2. Simulation of ultrasound systems: Field II
  - (a) Field II. Principles and overview
  - (b) Simulating B-mode imaging systems and blood flow
3. Non-linear ultrasound imaging
  - (a) Linear and Non-linear wave propagation
  - (b) Why is non-linear better?
4. Exercises
  - (a) Questions for exercises 3
  - (b) Questions for assignments

Reading material: JAJ, Chapters 2.5-6 and 4.2, Pages 27-44 and 70-75

2

## Discussion of time and phase shift systems

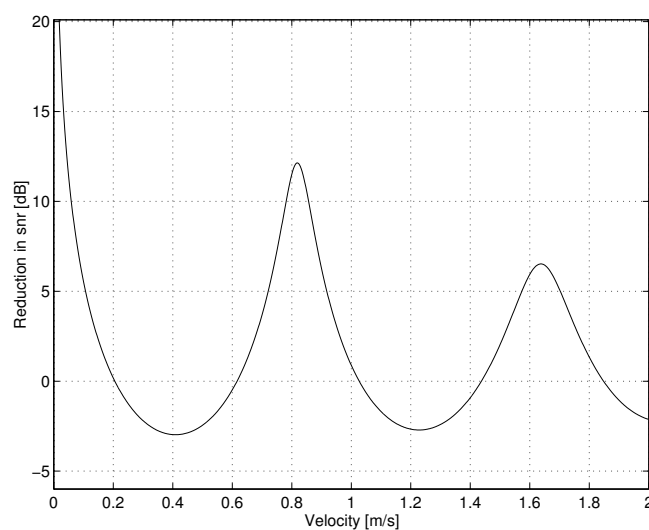
Calculate what you would get in a time and phase shift velocity estimation systems for the parameters given below.

Assume a peak velocity of 0.6 m/s at an angle of 60 degrees at the center of the vessel. The center frequency of the probe is 3 MHz, and the pulse repetition frequency is 3.2 kHz. The speed of sound is 1500 m/s.

1. How much is the time shift between two ultrasound pulse emissions?
2. What is the largest velocity detectable, if the cross-correlation function is calculated and searched over two wavelengths?
3. What is the highest detectable velocity for a phase shift system?
4. What is the loss in SNR for a velocity of 0.05 m/s based on Figure 7.5 and 8.3 for the two systems?

3

## Reduction in signal-to-noise ratio



$$f_0 = 3 \text{ MHz}$$

$$f_{prf} = 3.2 \text{ kHz}$$

$$B_r = 0.08$$

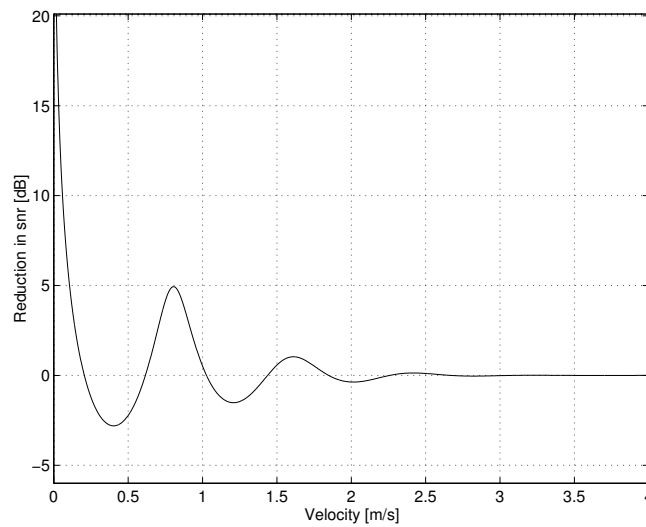
Center frequency of transducer

Pulse repetition frequency

Relative bandwidth of pulse

4

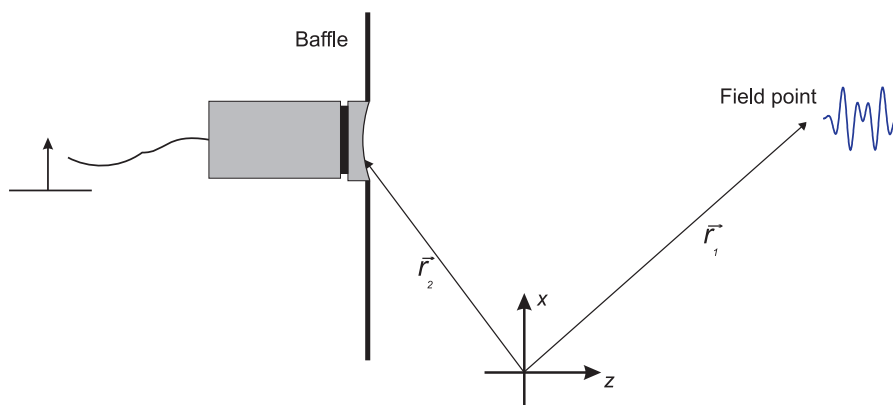
## Stationary echo canceling



Reduction of the signal-to-noise ratio due to the stationary echo canceling filter as a function of velocity. A Gaussian 3 MHz pulse with a relative bandwidth of 0.2 was used. The pulse repetition frequency was 3.2 kHz.

5

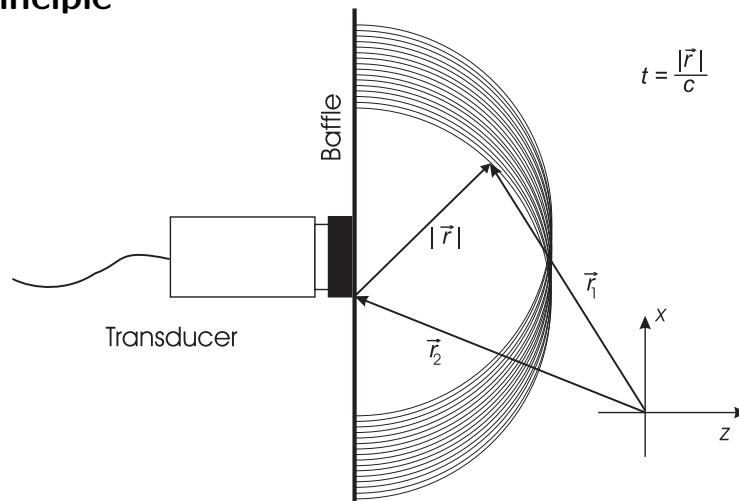
## Linear Acoustic System: Spatial impulse responses



Impulse response at a point in space.

6

## Huygens' Principle



Arrival times:  $t = |\vec{r}|/c$

Moving the point results in a new impulse response:

Spatial Impulse Responses -  $h(\vec{r}_1, t)$

7

## Ultrasound fields

Emitted field:

$$p(\vec{r}_1, t) = \rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1, t)$$

Pulse echo field:

$$v_r(\vec{r}_1, t) = v_{pe}(t) * f_m(\vec{r}_1) * h_{pe}(\vec{r}_1, t)$$

$$f_m(\vec{r}_1) = \frac{\Delta \rho(\vec{r}_1)}{\rho_0} - \frac{2 \Delta c(\vec{r}_1)}{c}$$

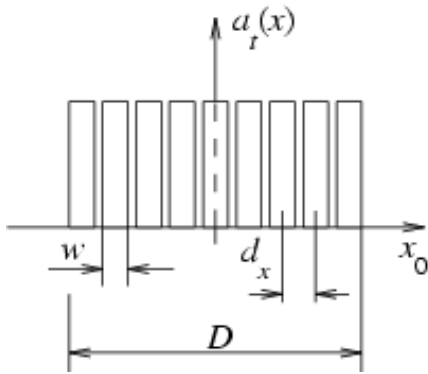
Continuous wave fields:

$$\mathcal{F}\{p(\vec{r}_1, t)\}, \quad \mathcal{F}\{v_r(\vec{r}_1, t)\}$$

All fields can be derived from the spatial impulse response.

8

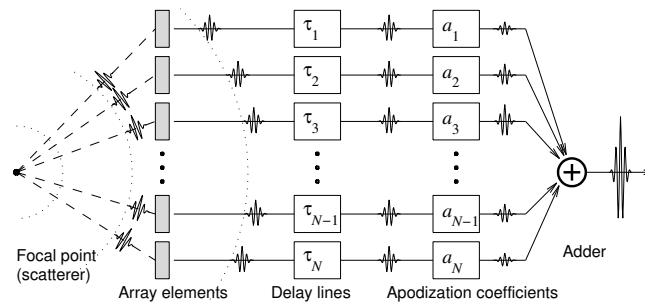
## Array geometry



- $d_x$  - Element pitch. For linear array:  $\approx \lambda = c/f_0$ , for phased array:  $\approx \lambda/2$
- $w$  - Width of element
- $k_e = d_x - w$  - Kerf (gap between elements)
- $D = (N_e - 1)d_x + w$  - Size of transducer
- Commercial 7 MHz linear array:
  - Elements:  $N_e = 192$ ,  
64 active at the same time
  - $\lambda = c/f_0 = 1540/7 \cdot 10^6 = 0.22$  mm
  - Pitch:  $d_x = 0.208$  mm
  - Width:  $D = 3.9$  cm
  - Height:  $h = 4.5$  mm
  - Kerf:  $k_e = 0.035$  mm

9

## Beamforming in Modern Scanners



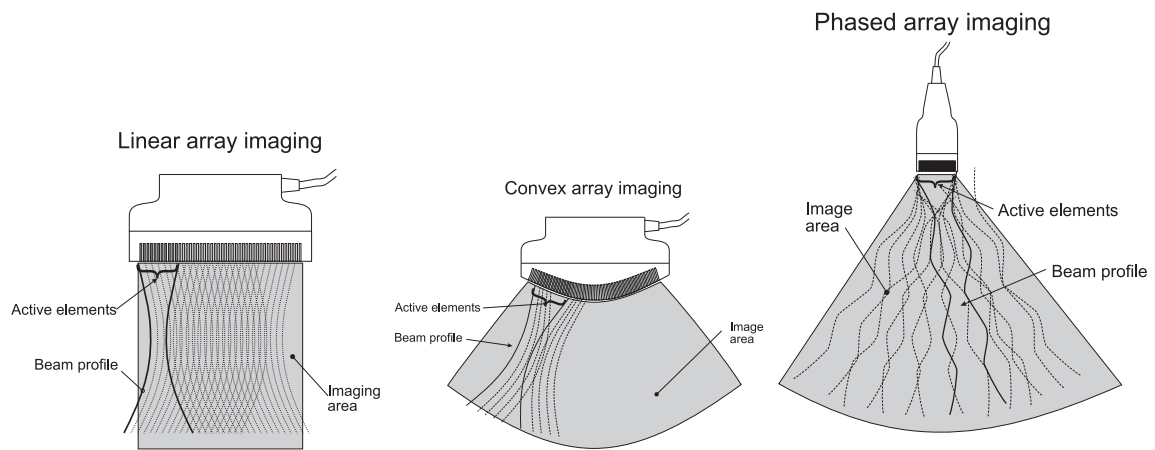
$$s(t) = \sum_1^{N_{xdc}} a_i y_i(t - \tau_i)$$

$$\tau_i = \frac{|\vec{r}_c - \vec{r}_f| - |\vec{r}_i - \vec{r}_f|}{c}$$

- $a_i$  - Weighting coefficient (apodization)
- $y_i(t)$  - Received signal
- $\vec{r} = [x, y, z]^T$  - Spatial position
- $\vec{r}_i$  - Position of transducer element,
- $\vec{r}_c$  - Beam reference point
- $\vec{r}_f$  - Focal point
- $c$  - Speed of sound

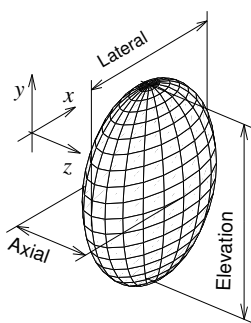
10

## Imaging methods

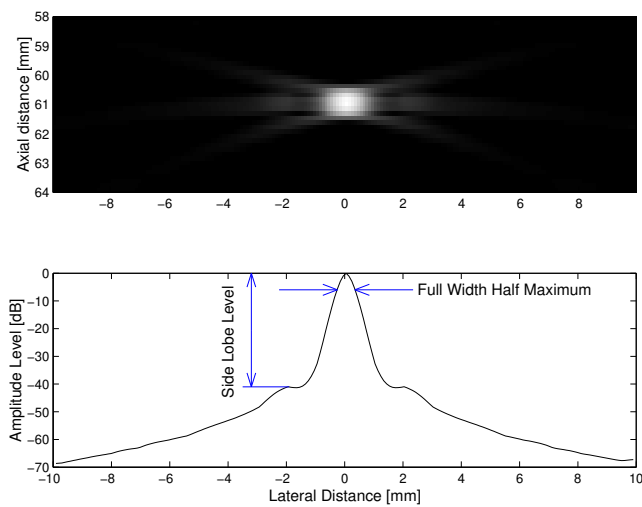


11

## PSF Characteristics



- The PSF is three dimensional
- The B-mode images are only 2-D
- Displayed on a logarithmic scale
- Maximum taken along  $z$
- Parameters used: FWHM, side- and grating-lobe level



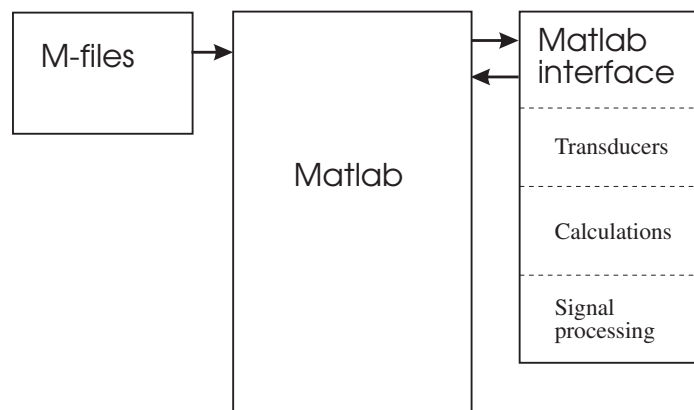
12

## Field II Simulation system

- Transducer modeled by dividing it into rectangles, triangles or bounding lines.
- C program interfaced to Matlab.
- Matlab used as front-end.
- Can handle any transducer geometry.
- Physical understanding of transducer.
- Pre-defined types: piston and concave single element, linear array, phased array, convex array, 2D matrix
- Any focusing, apodization, and excitation pulse.
- Multiple focusing and apodization.
- Dynamic focusing.
- Can calculate all types of fields (emitted, received, pulsed, CW)
- Can generate artificial ultrasound images (phased and linear array images with multiple receive and transmit foci).
- Data storage not necessary.
- Post-processing in Matlab.
- Versions for: Windows, Linux, Apple OS-X, Sun
- Free program at: <http://field-ii.dk/>

13

## Field II Program Organization



Makes it possible to use Matlab for signal processing and imaging

14

## Using the Field II program

```
% Start the system and initialize the path

path(path,'/home/jaj/programs/field_II/M_files')

% Initialize the field system

field_init

% Set parameters for transducer aperture

f0=5e6;           % Transducer center frequency [Hz]
fs=100e6;         % Sampling frequency [Hz]
c=1540;           % Speed of sound [m/s]
width=0.15/1000;  % Width of element
element_height=10/1000; % Height of element [m]
kerf=0.05/1000;   % Kerf [m]
focus=[0 0 60]/1000; % Fixed focal point [m]
N_elements=64;    % Number of physical elements

% Set the sampling frequency

set_sampling(fs);
```

15

## Using the Field II program 2 - PSF calculation

```
% Make the aperture for the response

aperture = xdc_linear_array (N_elements, width, element_height, kerf, 1, 1, focus);

% Set the excitation of the aperture

excitation=sin(2*pi*f0*(0:1/(fs):3/f0));
excitation=excitation.*hanning(max(size(excitation)))';
xdc_excitation (aperture, excitation);
xdc_impulse (aperture, excitation);

% Calculate for the points

x=(-10:0.25:10); % Lateral positions for field calculation
points=[ x; x*0; 20 + x*0]'/1000; % Point for calculation

disp('Finding response for array')
[p, t_start] = calc_hhp(aperture,aperture,points);
```

16



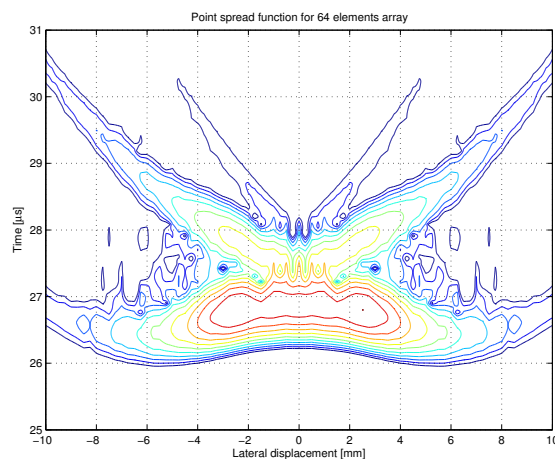
## Using the Field II program 2 - PSF calculation

```
% Find envelope and display

env=20*log10(abs(hilbert(p)));
env=env-max(max(env));
depth=((1:size(p,1))/fs + t_start)*c/2;
contour (x, depth*1000, env, -0.1:-6:-60)
colormap(jet)
xlabel('Lateral distance [mm]')
ylabel('Axial distance [mm]')
```

17

## Point spread function



Point spread function for 64 element linear array at 20 mm depth without apodization (6 dB between the contour lines)

18

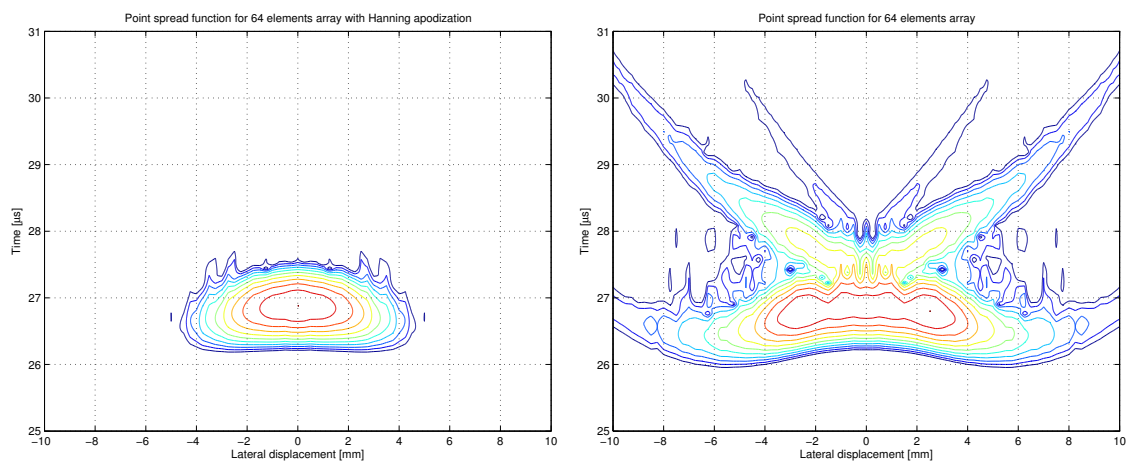
## Using the Field II program 2 - PSF calculation with apodization

```
% Make an other example with apodization  
  
xdc_apodization (aperture, 0, hanning(N_elements)');  
[p, t_start] = calc_hhp(aperture,aperture,points);
```

really quick to make different calculations.

19

## Point spread function with apodization



Point spread function for 64 element linear array at 20 mm depth with apodization (left) (6 dB between the contour lines) and without apodization (right)

20

## Realistic simulation of in-vivo imaging

Scattered field:

$$v_r(\vec{r}_1, t) = v_{pe}(t) * f_m(\vec{r}_1) * h_{pe}(\vec{r}_1, t)$$
$$f_m(\vec{r}_1) = \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c}$$

$\Delta\rho(\vec{r}_1)$  - Spatial variation in density

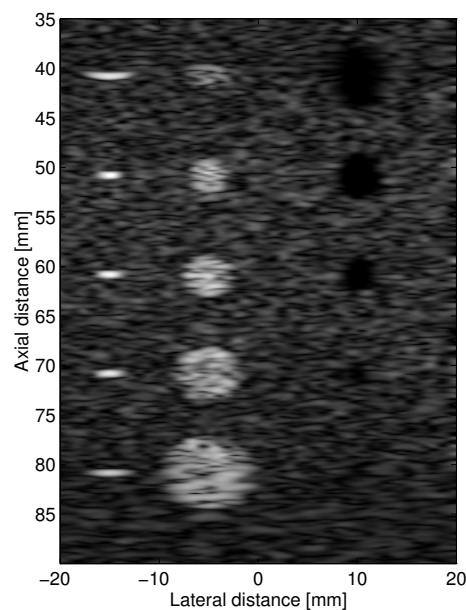
$\Delta c(\vec{r}_1)$  - Spatial variation in speed of sound

Description of spatial variation in backscattering from anatomic image:

$$\sigma_{f_m}(\vec{r}_1)$$

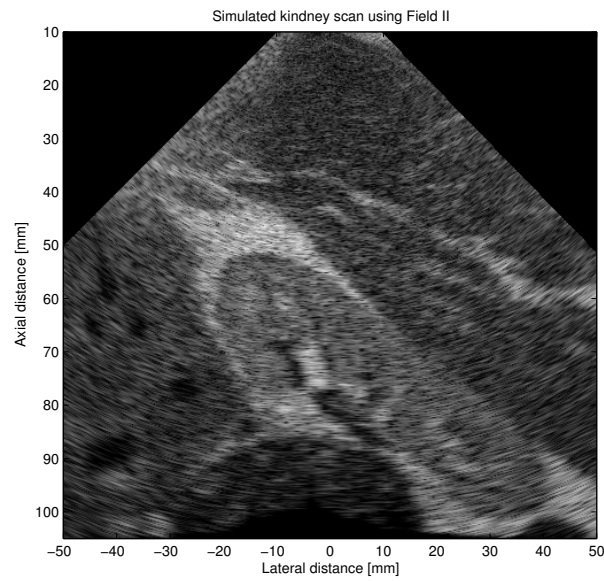
21

## Cyst phantom



22

## Simulation result for artificial kidney



Simulation can be done in parallel for multiple image lines

23

## Flow simulation in Field II

Pulse-echo model:

$$p_r(\vec{r}_1, t) = v_{pe}(t) \star_t f_m(\vec{r}_1) \star_r h_{pe}(\vec{r}_1, t),$$

The motion of the blood scatterers is modelled:

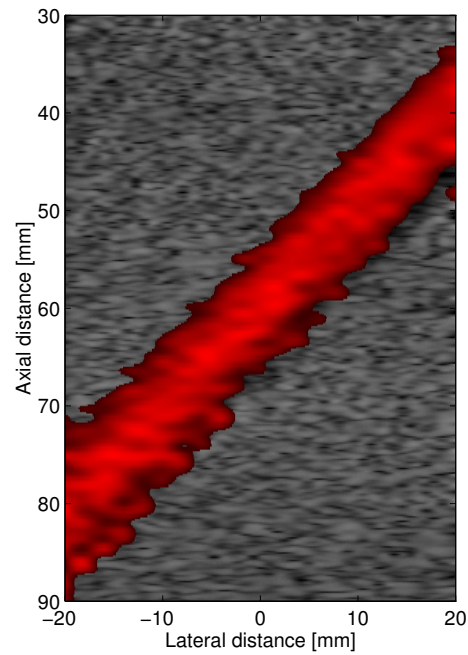
$$\vec{r}_1(i+1) = \vec{r}_1(i) + T_{prf} \vec{v}(\vec{r}_1(i), t)$$

Scatterers are propagated between pulses according to their velocity

$i$  - Emission number

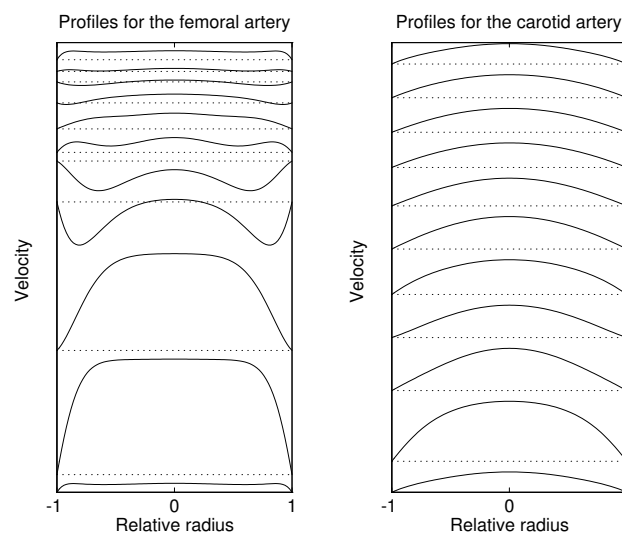
24

## Color flow imaging phantom



25

## Pulsating velocity profiles found in the human body

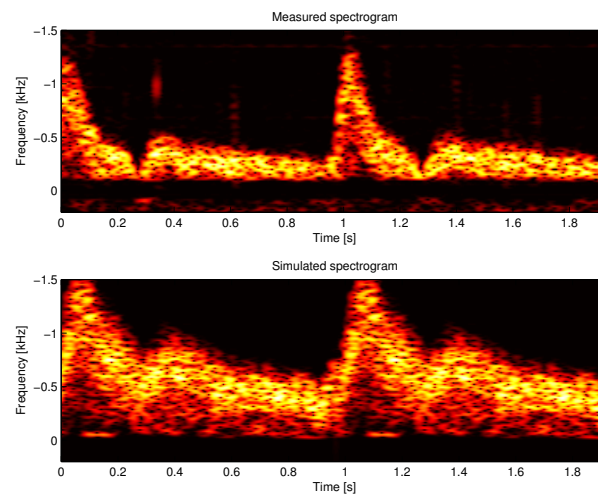


Velocity profiles from the common femoral (left) and carotid arteries (right) at different times in the cardiac cycle

The velocity data can be found on the web in the data directories

26

## Measured and simulated spectrograms for carotid artery



Data measured using a conventional B-K Medical 3535 scanner

27

## Field II Simulation system

- Any kind of transducer, excitation, impulse response, focusing and apodization can be simulated
- Simulations are done in C
- Scripting and pre- and post processing are done in Matlab
- All linear ultrasound imaging systems can be simulated including anatomic and flow systems
- Conventional and synthetic aperture systems can be simulated
- Simulations and measurements are accurate for both point spread functions, images, and flow modeling
- Simulations are easy to parallelize for shared disk, heterogeneous systems

28

## **Field II: How to get and use it**

- Citationware - Free to use, but you have to cite the papers
- J.A. Jensen: Field: A Program for Simulating Ultrasound Systems, Medical & Biological Engineering & Computing, pp. 351-353, Volume 34, Supplement 1, Part 1, 1996.
- J.A. Jensen and N. B. Svendsen: Calculation of pressure fields from arbitrarily shaped, apodized, and excited ultrasound transducers, IEEE Trans. Ultrason., Ferroelec., Freq. Contr., vol. 39, pp. 262-267, 1992.
- Web-site: <http://field-ii.dk/>

29

## **Non-linear ultrasound imaging**

- Linear ultrasound imaging
  - Non-linear wave propagation
  - Examples of non-linear waves
- Why is non-linear better?
  - Optimizing resolution
  - Pulse inversion imaging

Reading material: Chapter 2.5

30

## Linear Imaging

Depth in tissue:

$$D = \frac{ct}{2}$$

$c$  - Speed of sound (fixed at 1540 m/s)

$t$  - Time since pulse emission (200  $\mu$ s at 15 cm)

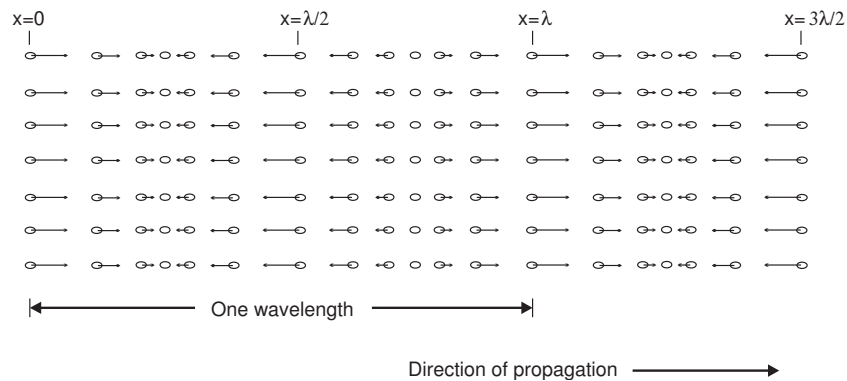
$$C = \sqrt{\frac{1}{\rho_0 \kappa}}$$

$\rho_0$  - Density of medium

$\kappa$  - Compressibility of medium

31

## Particle Displacement for Wave



32



## Non-linear Propagation

Speed of sound:

$$c(t) = c_0 \left( 1 + \frac{B}{2A} \cdot \frac{p(t)}{c_0 Z} \right)^{\left( \frac{2A}{B} + 1 \right)}$$

$B/A$  - Non-linearity parameter

$p(t)$  - Acoustic pressure

$c_0$  - Speed of sound for linear waves

$\rho_0$  - Density of the undisturbed medium

$Z$  - Characteristic acoustic impedance

$$A = \rho_0 c_0^2 \quad B = \rho_0 \left( \frac{\partial^2 p}{\partial \rho^2} \right) \quad Z = \rho_0 c_0$$

33

## B/A values

Medium	B/A
Water	4.2 – 6.1
Water (@ 30° C, 1 atm.)	5.2
Blood	6.3
Liver	7.6
Spleen	7.8
Fat	11.1

From Beyer (1974) and Bamber (1986)

34

## Example

Ultrasound wave, peak pressure of 1 MPa in water.

Maximum speed of sound:

$$c_{max} = c_0 \left( 1 + \frac{B}{2A} \frac{p(t)}{c_0 Z} \right)^{\left( \frac{2A}{B} + 1 \right)} = 1542.3 \text{ m/s}$$

Minimum speed:

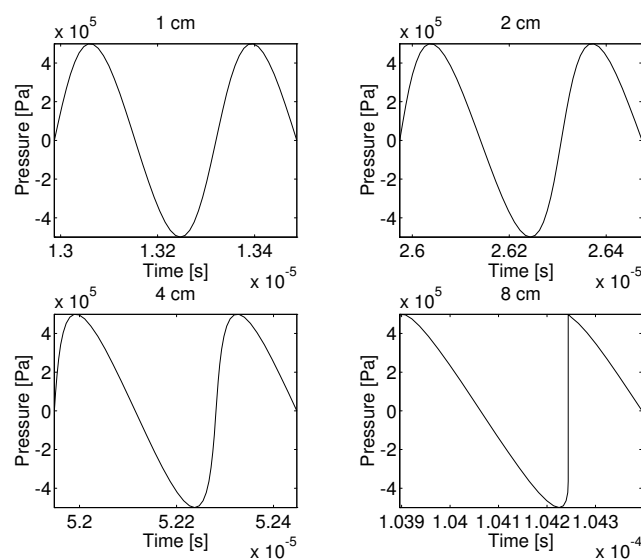
$$c_{min} = 1540 \left( 1 - \frac{5.2}{2} \frac{1 \cdot 10^6}{1540 \cdot 1.53 \cdot 10^6} \right)^{\left( \frac{2}{5.2} + 1 \right)} = 1537.7 \text{ m/s}$$

A span of 4.6 m/s.

For 15 cm depth, corresponding to  $100 \mu\text{s}$ :  $100 \cdot 10^6 \cdot 4.6 = 0.46 \text{ mm}$ .  
 $\lambda = 0.1 - 0.5 \text{ mm}$

35

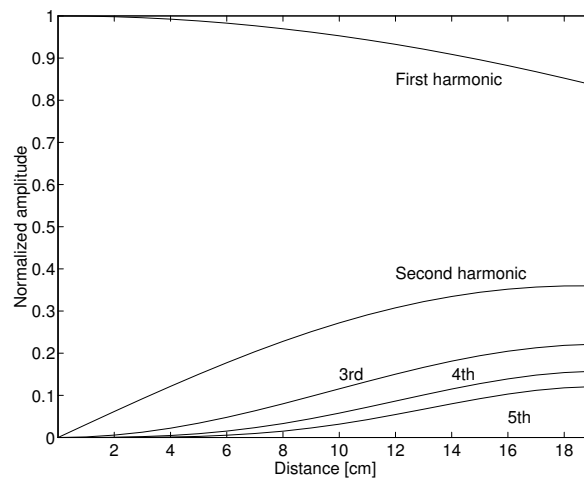
## Distortion of wave



Demo

36

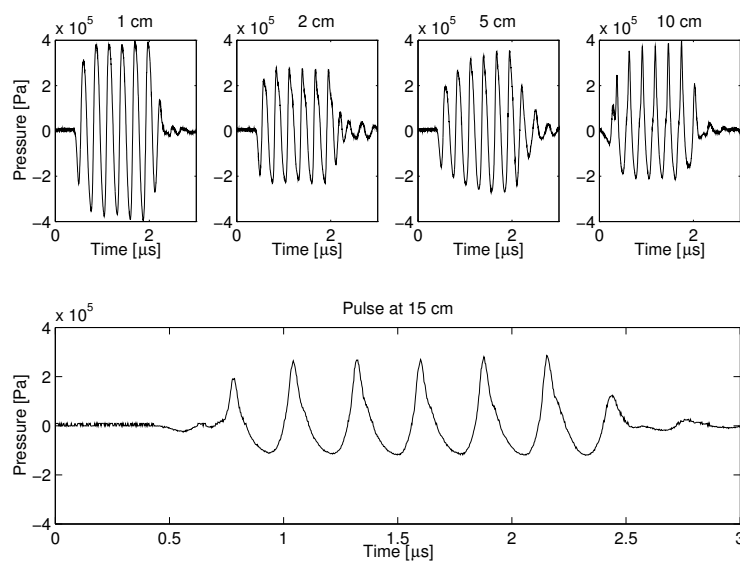
## Harmonic Example



Harmonics for a 1 MHz non-linear wave in water. Peak pressure is 1 MPa and  $B/A$  is 5.2.

37

## Measured Non-linear Pulses

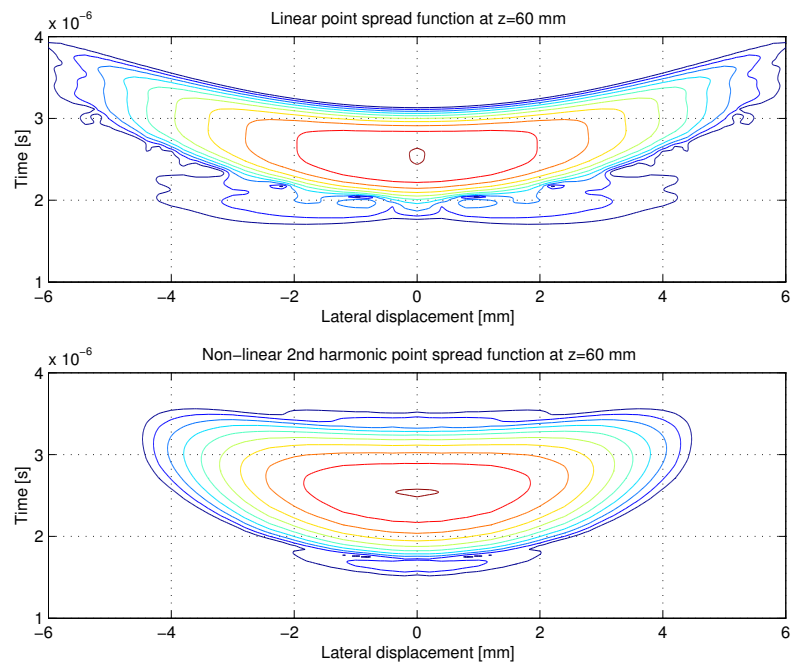


5 MHz phased array. Peak intensity  $1039 \text{ mW/cm}^2$ .

Matlab demo: non\_linear\_demo.m

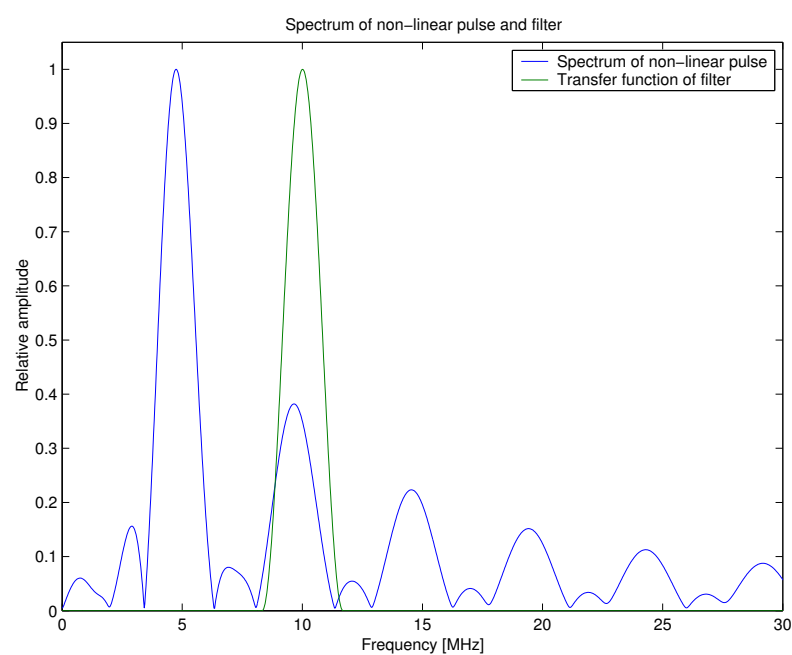
38

## What is the Advantage?



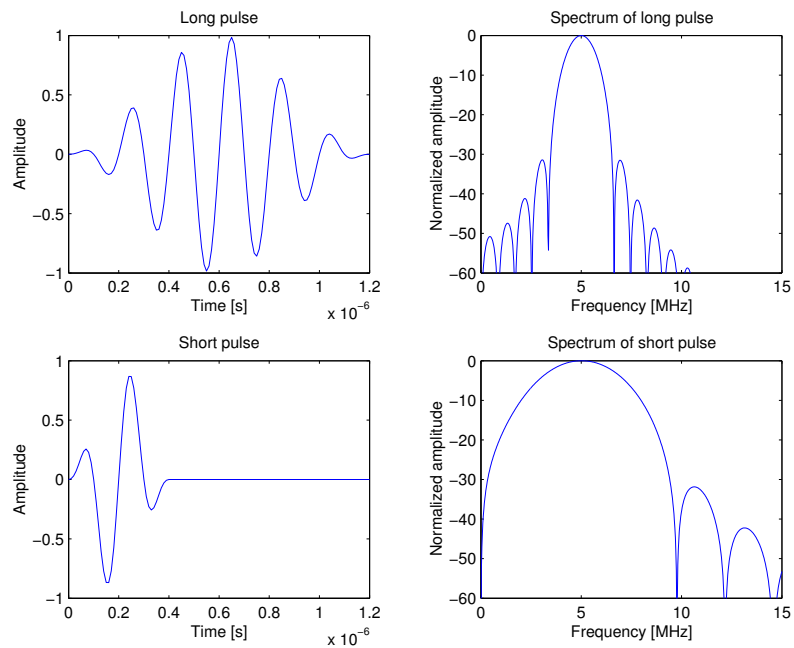
39

## Isolation of Second Harmonic



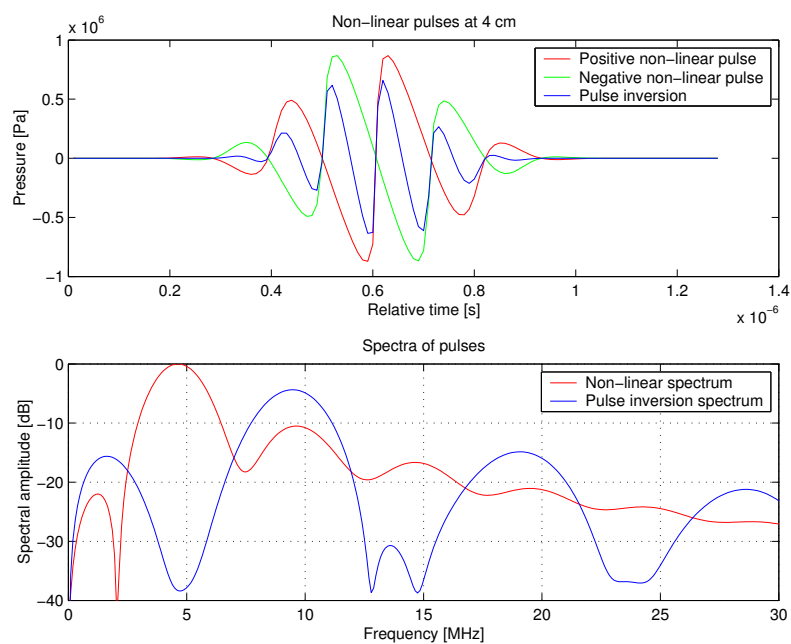
40

## Resolution and Bandwidth



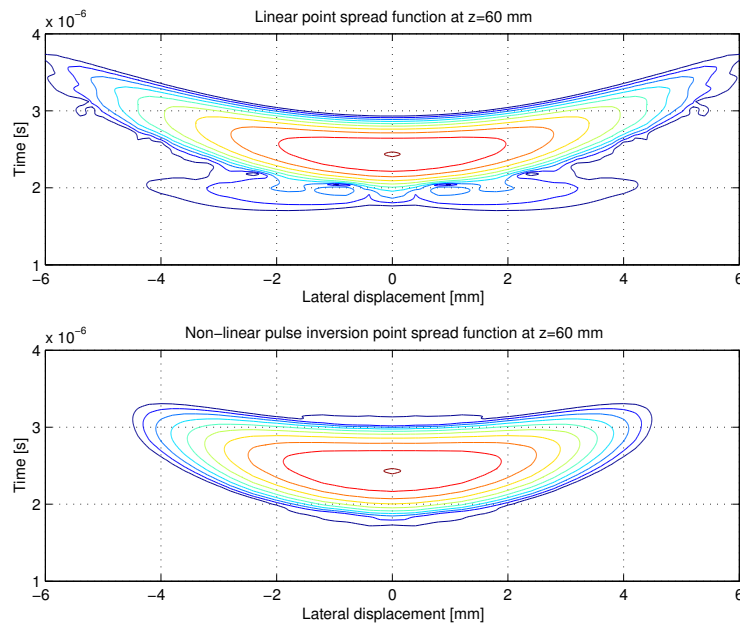
41

## Principle of Pulse Inversion



42

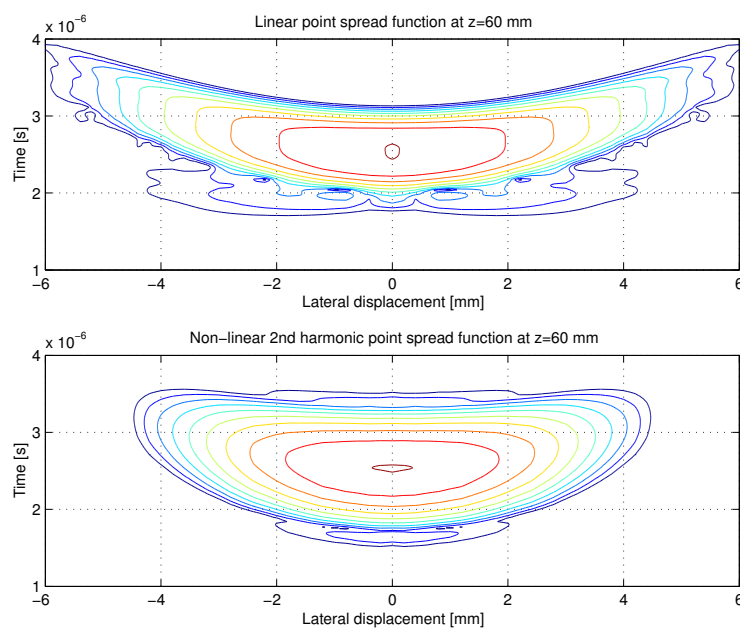
## PSF for Pulse Inversion



Matlab demo: non\_linear\_demo

43

## PSF without Pulse Inversion



44

## Summary

- Speed of sound is dependent on the pressure
- Positive pressure parts propagates faster than negative pressure parts
- Non-linear waves have lower side and grating lobes leading to images with higher contrast
- Pulse inversion gives higher axial resolution

45

## Exercise 3 about generating ultrasound RF flow data

Basic model, first emission:

$$r_1(t) = p(t) * s(t)$$

$s(t)$  - Scatterer amplitudes (white, random, Gaussian)

Second emission:

$$r_2(t) = p(t) * s(t - t_s) = r_1(t - t_s)$$

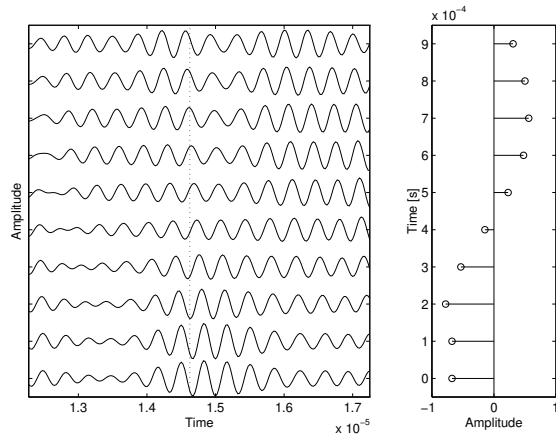
Time shift  $t_s$ :

$$t_s = \frac{2v_z}{c} T_{prf}$$

$r_1(t)$	Received voltage signal	$p(t)$	Ultrasound pulse
$*$	Convolution	$v_z$	Axial blood velocity
$c$	Speed of sound	$T_{prf}$	Time between pulse emissions

46

## A simple interpretation - a collection of scatterers



Collection of scatterers:

$$r_s(i) = - \sum_{k=1}^N a_k \sin\left(2\pi \frac{2v_z(k)}{c} f_0 T_{prf} i - \phi_k\right)$$

$$\phi_k = 2\pi f_0 \left(t_z - \frac{2d_k}{c}\right)$$

$k$  - Scatterer number

For a plug flow:

$$y_i(t) = p(t) * e(t - it_s) = y_0(t - it_s)$$

$$t_s = \frac{2v_z}{c} T_{prf}$$

For a sampled system:

$$y_i(n) = p(n) * e(n - i \cdot n_s) = y_0(n - i \cdot n_s)$$

$$n_s = \frac{2v_z}{c} T_{prf} f_s$$

Signal from a collection of scatterers crossing a beam from a concave transducer.

47

## Signal processing

1. Find ultrasound pulse (load from file)
2. Make scatterers
3. Generate a number of received RF signals
4. Study the generated signals
5. Compare with simulated and measured RF data

48