31545 Medical Imaging systems

Lecture 7: Velocity estimation using ultrasound

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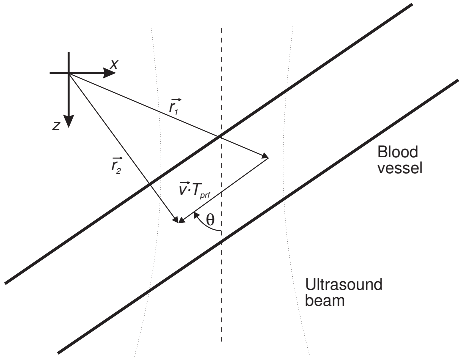
Topic of today: Velocity estimation for pulsed systems

1. Comments to previous lectures
2. Important concepts from last lecture
   (a) Model for ultrasound interaction with blood
3. Assignment on flow system
4. Pulsed wave ultrasound systems
   (a) Spectrum for a velocity distribution
   (b) Calculation of the spectrum
5. Color flow mapping systems
   (a) Phase shift estimator
6. Exercise 2 about point spread functions

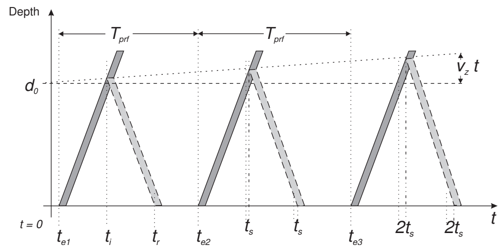
Reading material: JAJ, ch. 6 and 7, pages 113-148.

Basic measurement situation



$\vec{v}$	- Blood velocity	T_{prf}	- Time between pulse emissions
$\vec{r}_1$	- Position at first emission	$\vec{r}_2$	- Position at second emission
θ	- Angle between ultrasound beam and blood velocity		

Time-space diagram for a number of pulse emissions and receptions



Time shift between emissions:

$$t_s = \frac{2|\vec{v}|\cos\theta}{c}T_{prf} = \frac{2v_z}{c}T_{prf}$$

Received signals:

$$y_1(t) = a \cdot e(t - \frac{2d}{c})$$
$$y_2(t) = a \cdot e(t - \frac{2d}{c} - t_s) = y_1(t - t_s)$$

$\vec{v}$	Blood velocity	v_z	Blood velocity along ultrasound direction
T_{prf}	Time between pulse emissions	θ	Angle between ultrasound beam and velocity
$e(t)$	Emitted signal		

Model for the received signals (single scatterer)

First emission:

$$r_0(t) = a\sin(2\pi f_0(t_p - \frac{2d}{c}))$$

Second emission:

$$r_1(t) = a\sin(2\pi f_0(t_p - \frac{2d}{c} - t_s))$$

i'th emission:

$$r_i(t) = a\sin(2\pi f_0(t_p - \frac{2d}{c} - t_s i))$$

Final received signals (single scatterer)

Measurement at one fixed time t_z or depth:

$$\phi = 2\pi f_0(t_z - \frac{2d}{c})$$

gives

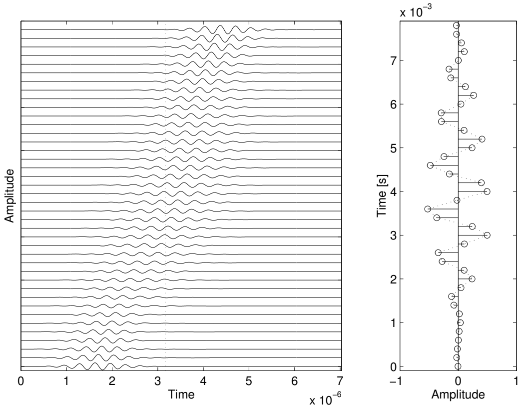
$$r_i(t_x) = -a\sin(2\pi f_0 t_s i - \phi) = -a\sin(2\pi \frac{2v_z}{c} f_0 T_{prf} i - \phi)$$

Frequency of sampled signal:

$$f_p = -\frac{2v_z}{c} f_0$$

v_z	Blood velocity along ultrasound direction	T_{prf}	Time between pulse emissions
f_0	Center frequency of transducer	c	Speed of sound
a	Scattering "strength"	t_p	Time relative to pulse emissions
t_z	Sampling time		

A simple interpretation - single scatterer



Signal from a single moving scatterer crossing a beam from a concave transducer.

Time shift between received signals:

$$t_s = \frac{2v_z}{c} T_{prf}$$

Received signal:

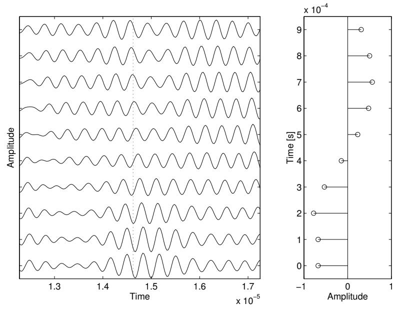
$$r_s(i) = -a\sin(2\pi \frac{2v_z}{c} f_0 T_{prf} i - \phi)$$

$$\phi = 2\pi f_0 \left(t_z - \frac{2d}{c}\right)$$

Frequency of received signal:

$$f_p = \frac{2v_z}{c} f_0$$

A simple interpretation - a collection of scatterers



Signal from a collection of scatterers crossing a beam from a concave transducer.

Collection of scatterers:

$$r_s(i) = -\sum_{k=1}^{N} a_k \sin(2\pi \frac{2v_z(k)}{c} f_0 T_{prf} i - \phi_k)$$

$$\phi_k = 2\pi f_0 \left(t_z - \frac{2d_k}{c}\right)$$

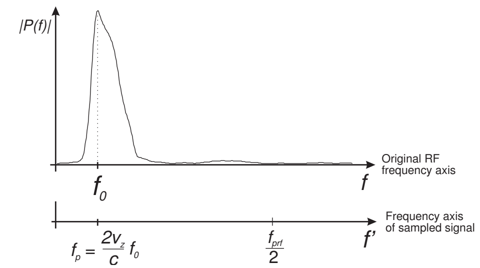
k - Scatterer number

For a plug flow:

$$y_i(t) = p(t) * e(t - it_s) = y_0(t - it_s)$$

$$t_s = \frac{2v_z}{c} T_{prf}$$

Frequency axis scaling



Spectrum of sampled signal for single scatterer moving at a velocity of v_z

Spectrum equals:

$$R(f) = P\left(\frac{c}{2v_z} f\right) * W(f)$$

$$W(f) = \frac{\sin \pi f N T_{prf}}{\sin \pi f T_{prf}} e^{-j\pi N T_{prf}}$$

Frequency scaled by:

$$\frac{2v_z}{c}$$

M	Number of cycles in pulse
$w(t)$	window due to sampling of a finite number of lines
$P(f)$	Spectrum of pulse

Physical effects

Down shift in center frequency due to attenuation:

$$\Delta f = \beta_1 B_r^2 f_0^2 d_0$$

Down shift in resulting pulsed wave spectrum:

$$\Delta f_{pw,att} = \frac{2v_z}{c} \cdot \beta_1 B_r^2 f_0^2 d_0,$$

Doppler shift due to the motion of the blood during the pulse's interaction:

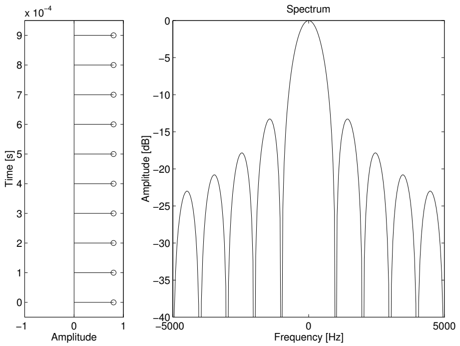
$$\Delta f_{pw,f_d} = \frac{2v_z}{c} \frac{2v_z}{c} f_0.$$

Non-linear components:

$$f_{\text{non-linear}} = \frac{2v_z}{c} f_{\text{har}}$$

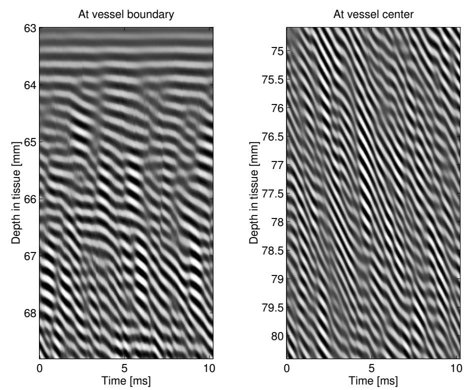
Bias depends on whether $|f_{\text{non-linear}}| > f_{prf}/2$ or not.

Spectrum for stationary signal



Signal obtained from stationary tissue and its spectrum.

RF signal for vessel with parabolic flow



x-direction: Time between pulse emission,
y-direction: Depth (time since one pulse emission)

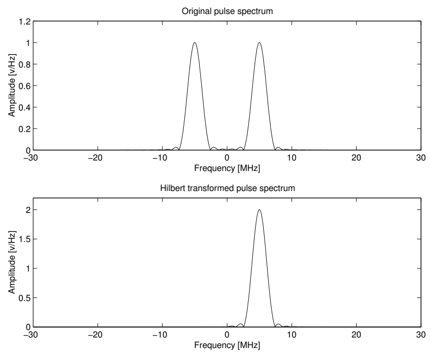
Discussion of assignment

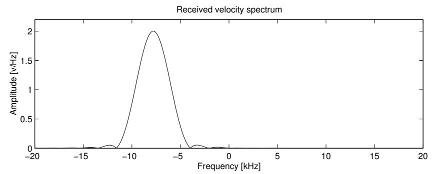
Calculate what you would get in a velocity estimation system for the time shift and the estimated frequency.

Assume a velocity of 0.75 m/s at an angle of 45 degrees. The center frequency of the probe is 3 MHz, and the pulse repetition frequency is 10 kHz. The speed of sound is 1500 m/s.

1. How much is the time shift between two ultrasound pulse emissions?
2. What would the center frequency of the received pulse wave spectrum be?
3. What is the highest velocity possible to estimate?
4. To what depth can this velocity be estimated?

Hilbert transformation

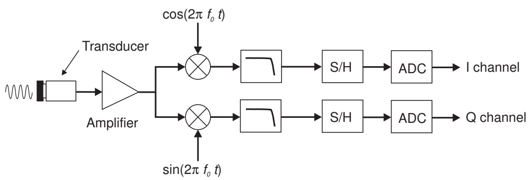




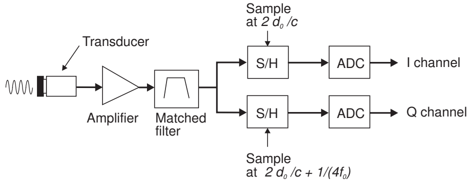
One sided spectrum created by Hilbert transforming the received signal. Thereby the sign of the frequency and velocity can be detected.

Pulsed wave systems using complex signals

Conventional analog demodulation

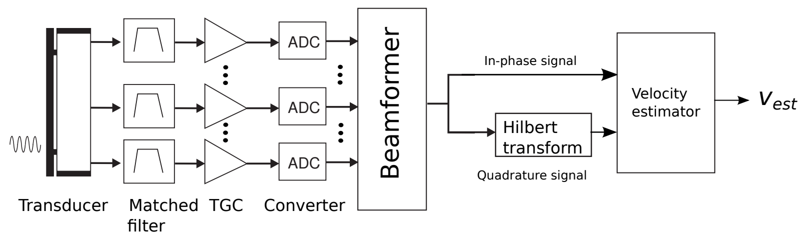


RF quadrature sampling

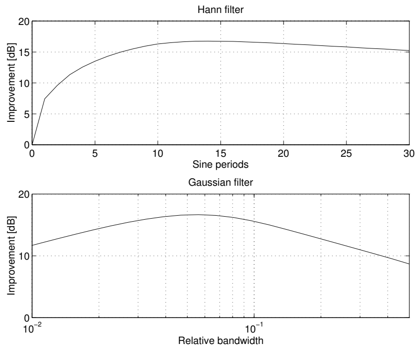


Modern digital pulsed wave systems using complex signals

RF sampling and Hilbert transformation



Effect of matched filter



Improvement in instantaneous signal power to mean noise power

Spectrum for a velocity distribution

Typical velocity distributions:

$$v(r) = v_0 \left[1 - \left(\frac{r}{R}\right)^{p_o}\right]$$

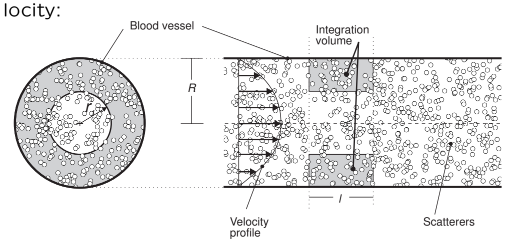
Parabolic flow for $p_o = 2$

Plug flow for $p_o \to \infty$

Frequencies received:

$$f_d(r) = \frac{2v_0 f_0}{c}\left[1 - \left(\frac{r}{R}\right)^{p_o}\right]\cos(\theta)$$

Power density spectrum is found by calculating the number of scatterers that move at a particular velocity:



Distribution of scatterers in a tube

r	radial position	R	radius of vessel
v_0	maximum velocity found at center of vessel		

Calculation of scatterer distribution

Total number of scattering particles with a velocity v less than v_1:

$$n_p(v < v_1) = \int_{r_1}^{R} l 2\pi r \rho_p dr = l\pi\rho_p(R^2 - r_1^2),$$

Particle density as a function of velocity:

$$p_v(v) = \frac{dn_p(v < v_1)}{dv} = l\pi\rho_p \frac{d(R^2 - r^2)}{dv} = -2l\pi\rho_p r\frac{dr}{dv}.$$

For parabolic flow profile:

$$r = R\left(1 - \frac{v}{v_0}\right)^{1/p_o}$$

$$\frac{dr}{dv} = -R\left(1 - \frac{v}{v_0}\right)^{\frac{1}{p_o}-1}\frac{1}{v_0 p_o}.$$

Combining gives:

$$p_v(v) = \frac{2lR^2\pi\rho_p}{p_o v_0}\frac{1}{\left(1 - \frac{v}{v_0}\right)^{1-\frac{2}{p_o}}}.$$

r	radial position,	R	radius of vessel
v_0	maximum velocity found at center of vessel	ρ_p	particle density
v_1	velocity at the radial position r_1		

Power density spectrum from scatterer distribution

Normalized power density:

$$G(f_d) = \begin{cases} \dfrac{2}{p_o \cdot f_{max}\left(1 - \frac{f_d}{f_{max}}\right)^{1-\frac{2}{p_o}}} & \text{for } 0 < f_d < f_{max} \\ 0 & \text{else} \end{cases}$$

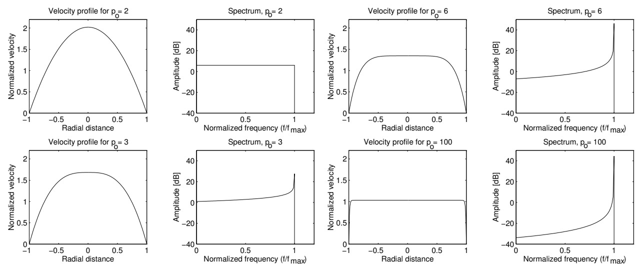
$$f_{max} = \frac{2v_0 f_0}{c}\cos(\theta).$$

Parabolic profile ($p_0 = 2$):

$$G(f_d) = \frac{1}{f_{max}} \quad \text{for } 0 < f_d < f_{max}$$

r	radial position	R	radius of vessel
v_0	maximum velocity found at center of vessel	ρ_p	particle density
v_1	velocity at the radial position r_1	f_0	ultrasound frequency
$\cos(\theta)$	angle between ultrasound beam and flow		

Examples of flow profiles and corresponding power density spectra

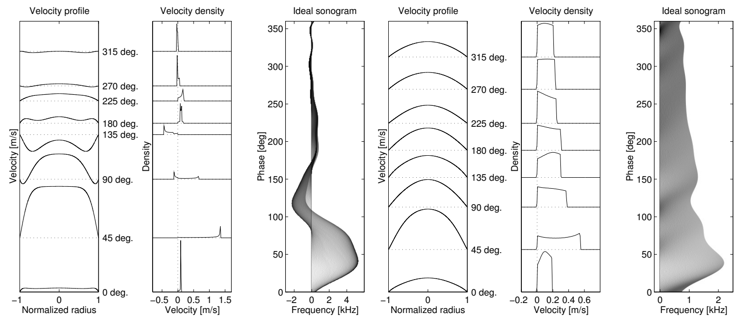


Idealized velocity profiles and corresponding normalized power density spectra.

Parabolic flow ($p_o = 2$) gives rectangular distribution of velocities and flat spectrum

Plug flow ($p_o \to \infty$) the spectrum approaches a monochromatic shape, because nearly all scatterers are moving at the same velocity.

Examples of flow profiles and corresponding power density spectra for flow in carotis and femoralis

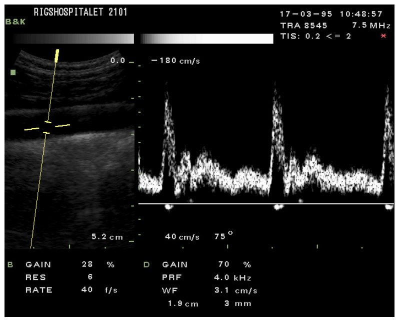


Series of velocity profiles for a common femoral artery (left) and common carotid artery (right) together with corresponding velocity densities and ideal sonograms. All curves are shown relative to the phase in the cardiac cycle.

Calculation of the velocity spectrum

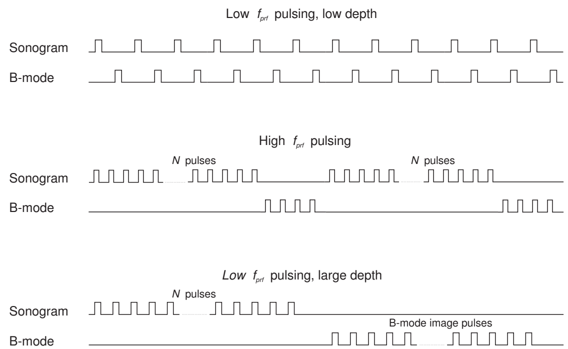
1. Sample RF signal from transducer and apply matched filter
2. Perform Hilbert transform and take out one sample per emission at range gate depth
3. Apply window on data and make a Fourier transform on the last 128 or 256 samples
4. Remove low-frequency samples from tissue (stationary echo canceling)
5. Compress data and display for a dynamic range of 40-60 dB as a time-velocity (frequency) plot
6. Repeat this process every 1-5 ms

This is the topic of exercise 4



Spectrogram from carotid artery

Emission sequences for duplex ultrasound scanning



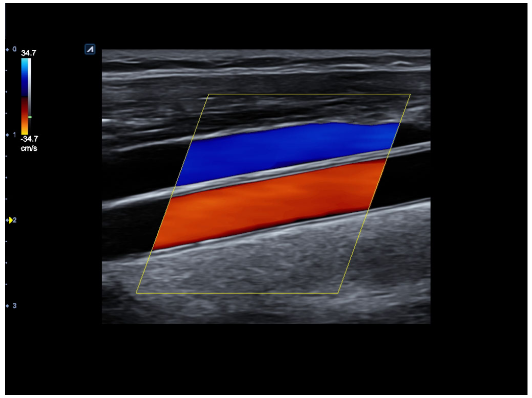
Pulsing strategies for different values of f_{prf} and depths in tissue.

Pulse wave ultrasound systems for velocity estimation

- Instantaneous Doppler shift not used, but shift in position between pulse
- Influence from different physical effects
- Description of pulsed wave system
- Finding the velocity direction
- Range/velocity ambiguity
- Spectrum for a random collection of scatterers

- Can only measure the velocity distribution at one place. Would be convenient with an image of velocity

Color flow map



Blood supply to and from the brain (Carotid artery and jugular vein)

Color flow mapping using phase shift estimation

Received demodulated signal:

$$\begin{aligned} r_{cfm}(i) &= a \cdot \exp(-j(2\pi \frac{2v_z}{c} f_0 i T_{prf} + \phi_f)) \\ &= a \cdot \exp(-j\phi(t)) = x(i) + jy(i) \end{aligned}$$

Velocity estimation:

$$\frac{d\phi}{dt} = \frac{d\left(-2\pi \frac{2v_z}{c} f_0 t + \phi\right)}{dt} = -2\pi \frac{2v_z}{c} f_0$$

Find the change is phase as a function of time gives quantity proportional to the velocity.

Realization

$$\begin{aligned} \tan(\Delta\phi) &= \tan\left(\arctan\left(\frac{y(i)}{x(i)}\right) - \arctan\left(\frac{y(i-1)}{x(i-1)}\right)\right) \\ &= \frac{\frac{y(i)}{x(i)} - \frac{y(i-1)}{x(i-1)}}{1 + \frac{y(i)}{x(i)} \cdot \frac{y(i-1)}{x(i-1)}} \\ &= \frac{y(i) \cdot x(i-1) - y(i-1)x(i)}{x(i)x(i-1) + y(i)y(i-1)} \end{aligned}$$

using that

$$\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}.$$

Then

$$\arctan\left(\frac{y(i)x(i-1) - y(i-1)x(i)}{x(i)x(i-1) + y(i)y(i-1)}\right) = -2\pi f_0 \frac{2v_z}{c} T_{prf}.$$

Color flow mapping using phase shift estimation

Using the complex autocorrelation:

$$R(m) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{i=-N}^{N} r^*_{cfm}(i) r_{cfm}(i+m),$$

Actual determination from the complex autocorrelation:

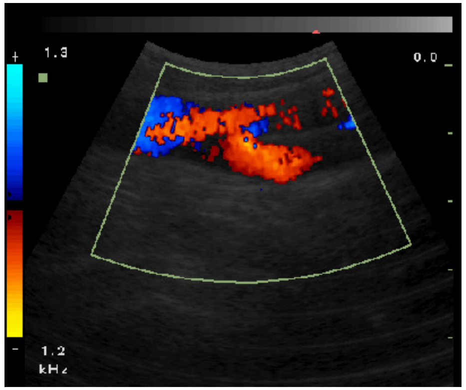
$$v_z = -\frac{c f_{prf}}{4\pi f_0} \arctan \left(\frac{\sum_{i=0}^{N_c-2} y(i+1)x(i) - x(i+1)y(i)}{\sum_{i=0}^{N_c-2} x(i+1)x(i) + y(i+1)y(i)} \right) = -\frac{c f_{prf}}{4\pi f_0} \arctan \left(\frac{\Im\{R(1)\}}{\Re\{R(1)\}} \right)$$

Corresponds to the mean angular frequency:

$$\bar{\omega} = \frac{\int_{-\infty}^{+\infty} \omega P(\omega) d\omega}{\int_{-\infty}^{+\infty} P(\omega) d\omega}$$

$P(\omega)$ is the power density spectrum of the received, demodulated signal.

Color flow map

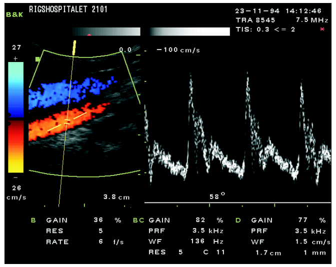


Blood supply to the leg (Root of femoral artery)

Video with CFM

Ultrasound systems for velocity estimation

- Instantaneous Doppler shift not used, but shift in position between pulse
- Influence from different physical effects minimized because of this
- Pulsed wave system described
- Spectrum for a random collection of scatterers described
- Color flow mapping system finds the velocity from the phase shift between emissions



- Stationary echo canceling and other methods for velocity estimation is the topic for the next lectures

Lecture next time

- Thursday: Cross correlation systems and stationary echo canceling
- Read Chapter 8 in JAJ
- Exercise 2 today Monday.
- Comments on Exercise 2

Discussion for next time

Calculate what you would get in a velocity estimation system for the phase shift and the power density spectrum for plug flow and parabolic flow.

Assume a peak velocity of 0.75 m/s at an angle of 45 degrees at the vessel center. The probe center frequency is 3 MHz, and the f_{prf} is 10 kHz. The speed of sound is 1500 m/s.

1. How much is the phase shift between two ultrasound pulse emissions when measuring the peak velocity?
2. What would the spectrum of the received signal be, if the velocity profile is parabolic?
3. What would the spectrum of the received signal be, if plug flow was found in the vessel?

Exercise 2 in generating ultrasound images

Basic model:

$$r(z,x) = p(z,x) * *s(z,x)$$

$r(z,x)$ - Received voltage signal (time converted to depth using the speed of sound)

$p(z,x)$ - 2D pulsed ultrasound field

$**$ - 2D convolution

$s(z,x)$ - Scatterer amplitudes (white, random)

z - Depth, x - Lateral distance

Signal processing

1. Find 2D ultrasound field (load from file)
2. Make scatterers with cyst hole
3. Make 2D convolution
4. Find compressed envelope data
5. Display the image
6. Compare with another pulsed field

Hint

Hint to make the scatterer map:

```
%  Make the scattere image

Nz=round(40/1000/dz);
Nx=round(40/1000/dx);
Nr=round(5/1000/dx);
e=randn (Nz, Nx);
x=ones(Nz,1)*(-Nx/2:Nx/2-1);
z=(-Nz/2:Nz/2-1)'*ones(1,Nx);
outside = sqrt(z.^2 + x.^2) > Nr*ones(Nz, Nx);
e=e.*outside;
```