31545 Medical Imaging systems

Lecture 7: Velocity estimation using ultrasound

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1. Important concepts from last lecture
   (a) Model for ultrasound interaction with blood

2. Assignment on flow system
   3. Pulsed wave ultrasound systems
      (a) Spectrum for a velocity distribution
      (b) Calculation of the spectrum

4. Color flow mapping systems
   (a) Phase shift estimator

5. Exercise 3 about generating ultrasound flow data

Reading material: JAJ, ch. 6 and 7, pages 113-148.
Final received signals (single scatterer)

Measurement at one fixed time \( t_z \) or depth:
\[
\phi = 2\pi f_0(t_z - \frac{2d}{c})
\]
gives
\[
r_s(t_s) = -a \sin(2\pi f_0 t_s i - \phi) = -a \sin(2\pi \frac{2v_z}{c} f_0 T_{prf} i - \phi)
\]

Frequency of sampled signal:
\[
f_p = \frac{2v_z}{c} f_0
\]

\( v_z \) Blood velocity along ultrasound direction
\( f_0 \) Center frequency of transducer
\( a \) Scattering "strength"
\( t_s \) Sampling time

A simple interpretation - single scatterer

\[
\phi = 2\pi f_0 \left( t_z - \frac{2d}{c} \right)
\]

Frequency scaled by:
\[
f_p = \frac{2v_z}{c} f_0
\]

Signal from a single moving scatterer crossing a beam from a concave transducer.

A simple interpretation - a collection of scatterers

\[
r_s(t) = \sum_{k=1}^{N} a_k \sin(2\pi \frac{2v_z}{c} f_0 T_{prf} i - \phi_k)
\]

\( \phi_k = 2\pi f_0 \left( t_z - \frac{2d}{c} \right) \)

\( k \) - Scatterer number

For a plug flow:
\[
y(t) = p(t) * e(t - it_i) = y_0(t - it_i)
\]
\[
t_s = \frac{2v_z}{c} T_{prf}
\]

A simple interpretation - a collection of scatterers

Signal from a collection of scatterers crossing a beam from a concave transducer.

Frequency axis scaling

Spectrum equals:
\[
R(f) = p \left( \frac{2v_z}{c} f_0 \right) * W(f)
\]
\[
W(f) = \frac{\sin \pi f T_{prf}}{\sin \pi f} e^{-\pi NT_{prf}}
\]

Frequency scaled by:
\[
f_p = \frac{2v_z}{c}
\]

Spectrum of sampled signal for single scatterer moving at a velocity of \( v_z \).

\( M \) Number of cycles in pulse
\( W(t) \) Window due to pulse emissions
\( P(f) \) Spectrum of pulse
Physical effects

Down shift in center frequency due to attenuation:
\[ \Delta f = \beta_1 B^2 f_0^2 d_0 \]

Down shift in resulting pulsed wave spectrum:
\[ \Delta f_{pw,att} = \frac{2v_z}{c} \beta_1 B^2 f_0^2 d_0, \]

Doppler shift due to the motion of the blood during the pulse’s interaction:
\[ \Delta f_{pw,f} = \frac{2v_z}{c} \frac{2v_z}{c} f_0. \]

Non-linear components:
\[ f_{non-linear} = \frac{2v_z}{c} f_{har} \]
Bias depends on whether \(|f_{non-linear}| > f_{prf}/2\) or not.

Discussion of assignment

Calculate what you would get in a velocity estimation system for the time shift and the estimated frequency.

Assume a velocity of 0.75 m/s at an angle of 45 degrees. The center frequency of the probe is 3 MHz, and the pulse repetition frequency is 10 kHz. The speed of sound is 1500 m/s.

1. How much is the time shift between two ultrasound pulse emissions?
2. What would the center frequency of the received pulse wave spectrum be?
3. What is the highest velocity possible to estimate?
4. To what depth can this velocity be estimated?
One sided spectrum created by Hilbert transforming the received signal. Thereby the sign of the frequency and velocity can be detected.

Modern digital pulsed wave systems using complex signals

RF sampling and Hilbert transformation

Effect of matched filter

Improvement in instantaneous signal power to mean noise power
Spectrum for a velocity distribution

Typical velocity distributions:

- **Parabolic flow for** $p = 2$
- **Plug flow for** $p \to \infty$

Frequencies received:

$$f_d(r) = \frac{2\nu_0 f_0}{c} \left[1 - \left(\frac{r}{R}\right)^2\right] \cos(\theta)$$

Parabolic profile ($p_0 = 2$):

$$G(f_d) = \frac{1}{f_{\text{max}}}$$

Normalized power density spectrum is found by calculating the number of scatterers that move at a particular velocity:

$$n_d(v < v_1) = \int_0^R 2\pi\rho_p dr = \pi\rho_p(R^2 - r_1^2)$$

Particle density as a function of velocity:

$$p_c(v) = \frac{dn_d(v < v_1)}{dv} = \pi\rho_p \frac{d(R^2 - r_1^2)}{dv} = -2\pi\rho_p \frac{dr}{dv}$$

For parabolic flow profile:

$$r = R \left(1 - \frac{v}{v_0}\right)^{1/p_0}$$

Combining gives:

$$p_c(v) = \frac{2\pi R^2 \rho_p}{p_0 v_0} \frac{1}{(1 - \frac{v}{v_0})^{1/p_0}}$$

Power density spectrum from scatterer distribution

Normalized power density:

$$G(f_d) = \begin{cases} \frac{2}{p_0 f_{\text{max}}(1 - \frac{f_d}{f_{\text{max}}})^{1/p_0}} & \text{for } 0 < f_d < f_{\text{max}} \\ 0 & \text{else} \end{cases}$$

$$f_{\text{max}} = \frac{2\nu_0 f_0}{c} \cos(\theta).$$

Examples of flow profiles and corresponding power density spectra

Idealized velocity profiles and corresponding normalized power density spectra.

Parabolic flow ($p_0 = 2$) gives rectangular distribution of velocities and flat spectrum.

Plug flow ($p_0 \to \infty$) the spectrum approaches a monochromatic shape, because nearly all scatterers are moving at the same velocity.
Examples of flow profiles and corresponding power density spectra for flow in carotis and femorals

Series of velocity profiles for a common femoral artery (left) and common carotid artery (right) together with corresponding velocity densities and ideal sonograms. All curves are shown relative to the phase in the cardiac cycle.

**Calculation of the velocity spectrum**

1. Sample RF signal from transducer and apply matched filter
2. Perform Hilbert transform and take out one sample per emission at range gate depth
3. Apply window on data and make a Fourier transform on the last 128 or 256 samples
4. Remove low-frequency samples from tissue (stationary echo canceling)
5. Compress data and display for a dynamic range of 40-60 dB as a time-velocity (frequency) plot
6. Repeat this process every 1-5 ms

This is the topic of exercise 4

**Emission sequences for duplex ultrasound scanning**

Pulsing strategies for different values of $f_{prf}$ and depths in tissue.

**Influence of beam and stochastic signal**

Central core of the vessel contributes to the spectrogram.

Effect of estimating the spectrogram from a stochastic signal.
Spectrogram from carotid artery

Computer simulation: snd_demo.m

Pulse wave ultrasound systems for velocity estimation

- Instantaneous Doppler shift not used, but shift in position between pulse
- Influence from different physical effects
- Description of pulsed wave system
- Finding the velocity direction
- Range/velocity ambiguity
- Spectrum for a random collection of scatterers

- Can only measure the velocity distribution at one place. Would be convenient with an image of velocity

- The topic after the break

Color flow map

Blood supply to and from the brain (Carotid artery and jugular vein)

Color flow mapping using phase shift estimation

Received demodulated signal:

\[ r_{cfm}(i) = a \cdot \exp(-j(2\pi^2v_z/c f_0 t_{prf} + \phi_f)) \]

\[ = a \cdot \exp(-j\phi(i)) = x(i) + jy(i) \]

Velocity estimation:

\[ \frac{d\phi}{dt} = \frac{d( -2\pi^2v_z/c f_0 t + \phi)}{dt} = -2\pi^2v_z/c f_0 \]

Find the change is phase as a function of time gives quantity proportional to the velocity.
Realization

$$\tan(\Delta \phi) = \tan \left( \arctan \left( \frac{y(i)}{x(i)} - \arctan \left( \frac{y(i-1)}{x(i-1)} \right) \right) \right)$$

$$= \frac{\frac{y(i)}{x(i)} - \frac{y(i-1)}{x(i-1)}}{1 + \frac{\frac{y(i)}{x(i)}}{\frac{y(i-1)}{x(i-1)}}}$$

$$= \frac{y(i) \cdot x(i-1) - y(i-1) \cdot x(i)}{x(i) \cdot x(i-1) + y(i) \cdot y(i-1)}$$

using that

$$\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A) \cdot \tan(B)}$$

Then

$$\arctan \left( \frac{y(i) \cdot x(i-1) - y(i-1) \cdot x(i)}{x(i) \cdot x(i-1) + y(i) \cdot y(i-1)} \right) = -2\pi f_0 \frac{2v_z}{c} T_{prf}.$$
Color flow map

Blood supply to the leg (Root of femoral artery)

Video with CFM

Ultrasound systems for velocity estimation

- Instantaneous Doppler shift not used, but shift in position between pulse
- Influence from different physical effects minimized because of this
- Pulsed wave system described
- Spectrum for a random collection of scatterers described
- Color flow mapping system finds the velocity from the phase shift between emissions
- Stationary echo canceling and other methods for velocity estimation is the topic for the next lectures

Lectures next time

- Thursday: Cross correlation systems and stationary echo canceling
- Read Chapter 8 in JAJ
- Exercise 3 today Monday.
- Hand-out of ultrasound assignment - Are groups correct on the web site?
- Comments on Exercise 3

Discussion for next time

Calculate what you would get in a velocity estimation system for the phase shift and the power density spectrum for plug flow and parabolic flow.

Assume a peak velocity of 0.75 m/s at an angle of 45 degrees at the center of the vessel. The center frequency of the probe is 3 MHz, and the pulse repetition frequency is 10 kHz. The speed of sound is 1500 m/s.

1. How much is the phase shift between two ultrasound pulse emissions when measuring the peak velocity?

2. What would the spectrum of the received signal be, if the velocity profile is parabolic?

3. What would the spectrum of the received signal be, if plug flow was found in the vessel?
Exercise 3 about generating ultrasound RF flow data

Basic model, first emission:

\[ r_1(t) = p(t) * s(t) \]

\( s(t) \) - Scatterer amplitudes (white, random, Gaussian)

Second emission:

\[ r_2(t) = p(t) * s(t - t_s) = r_1(t - t_s) \]

Time shift \( t_s \):

\[ t_s = \frac{2v_z T_{prf}}{c} \]

**Signal processing**

1. Find ultrasound pulse (load from file)
2. Make scatterers
3. Generate a number of received RF signals
4. Study the generated signals
5. Compare with simulated and measured RF data

| \( r_1(t) \) | Received voltage signal |
| \( p(t) \) | Ultrasound pulse |
| \( s(t) \) | Convolution |
| \( v_z \) | Axial blood velocity |
| \( c \) | Speed of sound |
| \( T_{prf} \) | Time between pulse emissions |