31545 Medical Imaging systems

Lecture 7: Velocity estimation using ultrasound

Jørgen Arendt Jensen Department of Health Technology Section for Ultrasound and Biomechanics Technical University of Denmark

September 18, 2023

Topic of today: Velocity estimation for pulsed systems

- 1. Comments to previous lectures
- 2. Important concepts from last lecture
 - (a) Model for ultrasound interaction with blood
- 3. Assignment on flow system
- 4. Pulsed wave ultrasound systems
 - (a) Spectrum for a velocity distribution
 - (b) Calculation of the spectrum
- 5. Color flow mapping systems
 - (a) Phase shift estimator
- 6. Exercise 2 about point spread functions

Reading material: JAJ, ch. 6 and 7, pages 113-148.





Model for the received signals (single scatterer)

First emission:

$$r_0(t) = a\sin(2\pi f_0(t_p - \frac{2d}{c}))$$

Second emission:

$$r_1(t) = a \sin(2\pi f_0(t_p - \frac{2d}{c} - t_s))$$

i'th emission:

$$r_i(t) = a\sin(2\pi f_0(t_p - \frac{2d}{c} - t_s i))$$

-	1	٠
-	4	,

Final received signals (single scatterer)

Measurement at one fixed time t_z or depth:

$$\phi = 2\pi f_0(t_z - \frac{2d}{c})$$

gives

$$r_i(t_x) = -a\sin(2\pi f_0 t_s i - \phi) = -a\sin(2\pi \frac{2v_z}{c} f_0 T_{prf} i - \phi)$$

Frequency of sampled signal:

$$f_p = -\frac{2v_z}{c}f_0$$

 v_z Blood velocity along ultrasound direction T_{prf} Time between pulse emissions f_0 Center frequency of transducer ca Scattering "strength" t_p t_z Sampling time

Speed of sound

Time relative to pulse emissions







Physical effects

Down shift in center frequency due to attenuation:

$$\Delta f = \beta_1 B_r^2 f_0^2 d_0$$

Down shift in resulting pulsed wave spectrum:

$$\Delta f_{pw,att} = \frac{2v_z}{c} \cdot \beta_1 B_r^2 f_0^2 d_0,$$

Doppler shift due to the motion of the blood during the pulse's interaction:

$$\Delta f_{pw,f_d} = \frac{2v_z}{c} \frac{2v_z}{c} f_0.$$

Non-linear components:

$$f_{\text{non-linear}} = \frac{2v_z}{c} f_{\text{har}}$$

Bias depends on whether $|f_{non-linear}| > f_{prf}/2$ or not.





Discussion of assignment

Calculate what you would get in a velocity estimation system for the time shift and the estimated frequency.

Assume a velocity of 0.75 m/s at an angle of 45 degrees. The center frequency of the probe is 3 MHz, and the pulse repetition frequency is 10 kHz. The speed of sound is 1500 m/s.

- 1. How much is the time shift between two ultrasound pulse emissions?
- 2. What would the center frequency of the received pulse wave spectrum be?
- 3. What is the highest velocity possible to estimate?
- 4. To what depth can this velocity be estimated?











Calculation of scatterer distribution

Total number of scattering particles with a velocity v less than v_1 :

$$n_p(v < v_1) = \int_{r_1}^{R} l 2\pi r \rho_p dr = l \pi \rho_p (R^2 - r_1^2),$$

Particle density as a function of velocity:

$$p_{v}(v) = \frac{dn_{p}(v < v_{1})}{dv} = l\pi\rho_{p}\frac{d(R^{2} - r^{2})}{dv} = -2l\pi\rho_{p}r\frac{dr}{dv}.$$

For parabolic flow profile:

$$r = R \left(1 - \frac{v}{v_0}\right)^{1/p_o}$$
$$\frac{dr}{dv} = -R \left(1 - \frac{v}{v_0}\right)^{\frac{1}{p_o} - 1} \frac{1}{v_0 p_o}$$

Combining gives:

$$p_v(v) = rac{2lR^2 \pi
ho_p}{p_o v_0} rac{1}{\left(1 - rac{v}{v_0}
ight)^{1 - rac{2}{p_o}}}$$

r radial position,

R radius of vessel

 v_0 maximum velocity found at center of vessel ho_p particle density

 v_1 velocity at the radial position r_1

19

Power density spectrum from scatterer distribution

Normalized power density:

$$G(f_d) = \begin{cases} \frac{2}{p_o \cdot f_{max} \left(1 - \frac{f_d}{f_{max}}\right)^{1 - \frac{2}{p_o}}} & \text{for } 0 < f_d < f_{max} \\ 0 & \text{else} \end{cases}$$
$$f_{max} = \frac{2v_0 f_0}{c} \cos(\theta).$$

Parabolic profile ($p_0 = 2$):

$$G(f_d) = \frac{1}{f_{max}} \quad \text{for } 0 < f_d < f_{max}$$

 $\begin{array}{cccc} r & \mbox{radial position} & R & \mbox{r} \\ v_0 & \mbox{maximum velocity found at center of vessel} & \rho_p & \mbox{g} \\ v_1 & \mbox{velocity at the radial position } r_1 & f_0 & \mbox{u} \\ \cos(\theta) & \mbox{angle between ultrasound beam and flow} \end{array}$

R radius of vessel

 ho_p particle density

 f_0 ultrasound frequency



Examples of flow profiles and corresponding power density spectra



Examples of flow profiles and corresponding power density spectra for flow in carotis and femoralis Ideal sonogram Ideal sonogram Velocity profile Velocity density Velocity profile Velocity density 350 350 315 dea 315 deg. 300 300 270 deg. 270 dea 225 deg. 250 250 225 deg. 180 deg. 135 deg "/s] ခြီ 200 ခြို 200 180 deg. -≩ /elocity Č ٩ 4 150 습 150 135 deg. 90 deg. 90 deg 100 100 45 deg. 45 deg 50 50 0 deg 0) deg. 1 0 Normalized radius -0.5 0 0.5 1 1.5 Velocity [m/s] -2 0 2 4 Frequency [kHz] 1 0 Normalized radius -0.2 0 0.2 0.4 0.6 Velocity [m/s] 0 1 2 Frequency [kHz]

Series of velocity profiles for a common femoral artery (left) and common carotid artery (right) together with corresponding velocity densities and ideal sonograms. All curves are shown relative to the phase in the cardiac cycle.

Calculation of the velocity spectrum

- 1. Sample RF signal from transducer and apply matched filter
- 2. Perform Hilbert transform and take out one sample per emission at range gate depth
- 3. Apply window on data and make a Fourier transform on the last 128 or 256 samples
- 4. Remove low-frequency samples from tissue (stationary echo canceling)
- 5. Compress data and display for a dynamic range of 40-60 dB as a timevelocity (frequency) plot
- 6. Repeat this process every 1-5 ms

This is the topic of exercise 4



Spectrogram from carotid artery



Pulse wave ultrasound systems for velocity estimation

- Instantaneous Doppler shift not used, but shift in position between pulse
- Influence from different physical effects
- Description of pulsed wave system
- Finding the velocity direction
- Range/velocity ambiguity
- Spectrum for a random collection of scatterers



• Can only measure the velocity distribution at one place. Would be convenient with an image of velocity

<section-header><section-header><section-header><text><text><page-footer>

Color flow mapping using phase shift estimation

Received demodulated signal:

$$r_{cfm}(i) = a \cdot \exp(-j(2\pi \frac{2v_z}{c}f_0iT_{prf} + \phi_f))$$

= $a \cdot \exp(-j\phi(t)) = x(i) + jy(i)$

Velocity estimation:

$$\frac{d\phi}{dt} = \frac{d\left(-2\pi\frac{2v_z}{c}f_0t + \phi\right)}{dt} = -2\pi\frac{2v_z}{c}f_0$$

Find the change is phase as a function of time gives quantity proportional to the velocity.

Realization $\tan(\Delta\phi) = \tan\left(\arctan\left(\frac{y(i)}{x(i)}\right) - \arctan\left(\frac{y(i-1)}{x(i-1)}\right)\right)$ $= \frac{\frac{y(i)}{x(i)} - \frac{y(i-1)}{x(i-1)}}{1 + \frac{y(i)}{x(i)} \cdot \frac{y(i-1)}{x(i-1)}}$ $= \frac{y(i) \cdot x(i-1) - y(i-1)x(i)}{x(i)x(i-1) + y(i)y(i-1)}$ using that $\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}.$ Then

$$\arctan\left(\frac{y(i)x(i-1) - y(i-1)x(i)}{x(i)x(i-1) + y(i)y(i-1)}\right) = -2\pi f_0 \frac{2v_z}{c} T_{prf}.$$

Color flow mapping using phase shift estimation

Using the complex autocorrelation:

$$R(m) = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{i=-N}^{N} r_{cfm}^{*}(i) r_{cfm}(i+m),$$

Actual determination from the complex autocorrelation:

$$v_{z} = -\frac{cf_{prf}}{4\pi f_{0}} \arctan\left(\frac{\sum_{i=0}^{N_{c}-2} y(i+1)x(i) - x(i+1)y(i)}{\sum_{i=0}^{N_{c}-2} x(i+1)x(i) + y(i+1)y(i)}\right) = -\frac{cf_{prf}}{4\pi f_{0}} \arctan\left(\frac{\Im\{R(1)\}}{\Re\{R(1)\}}\right)$$

Corresponds to the mean angular frequency:

$$\bar{\omega} = \frac{\int_{-\infty}^{+\infty} \omega P(\omega) d\omega}{\int_{-\infty}^{+\infty} P(\omega) d\omega}$$

 $P(\omega)$ is the power density spectrum of the received, demodulated signal.



Ultrasound systems for velocity estimation

- Instantaneous Doppler shift not used, but shift in position between pulse
- Influence from different physical effects minimized because of this
- Pulsed wave system described
- Spectrum for a random collection of scatterers described
- Color flow mapping system finds the velocity from the phase shift between emissions



• Stationary echo canceling and other methods for velocity estimation is the topic for the next lectures

Lecture next time

- Thursday: Cross correlation systems and stationary echo canceling
- Read Chapter 8 in JAJ
- Exercise 2 today Monday.
- Comments on Exercise 2

Discussion for next time

Calculate what you would get in a velocity estimation system for the phase shift and the power density spectrum for plug flow and parabolic flow.

Assume a peak velocity of 0.75 m/s at an angle of 45 degrees at the vessel center. The probe center frequency is 3 MHz, and the f_{prf} is 10 kHz. The speed of sound is 1500 m/s.

- 1. How much is the phase shift between two ultrasound pulse emissions when measuring the peak velocity?
- 2. What would the spectrum of the received signal be, if the velocity profile is parabolic?
- 3. What would the spectrum of the received signal be, if plug flow was found in the vessel?

33

Exercise 2 in generating ultrasound images

Basic model:

$$r(z,x) = p(z,x) * *s(z,x)$$

r(z,x) - Received voltage signal (time converted to depth using the speed of sound)

p(z,x) - 2D pulsed ultrasound field

** - 2D convolution

s(z, x) - Scatterer amplitudes (white, random)

z - Depth, x - Lateral distance

Signal processing

1. Find 2D ultrasound field (load from file)

2. Make scatterers with cyst hole

3. Make 2D convolution

4. Find compressed envelope data

5. Display the image

6. Compare with another pulsed field

Hint

Hint to make the scatterer map:

```
% Make the scattere image
```

```
Nz=round(40/1000/dz);
Nx=round(40/1000/dx);
Nr=round(5/1000/dx);
e=randn (Nz, Nx);
x=ones(Nz,1)*(-Nx/2:Nx/2-1);
z=(-Nz/2:Nz/2-1)'*ones(1,Nx);
outside = sqrt(z.^2 + x.^2) > Nr*ones(Nz, Nx);
e=e.*outside;
```