31545 Medical Imaging systems

Lecture 6: Interaction between flowing blood and ultrasound

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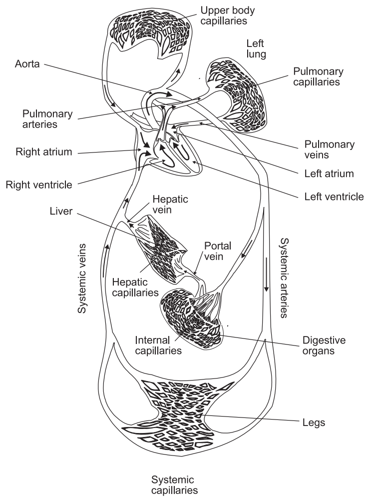
Topic of today: Interaction between blood and ultrasound

1. Important concepts from last lecture
2. Assignment: design parameters for blood velocity estimation system
3. Scattering of ultrasound
4. Ultrasounds interaction with flowing blood
5. Derivation of a model
6. Consequences of the model
7. Pulsed wave ultrasound systems

8. Exercise 2 about generating an ultrasound speckle image
9. Exercise 3 about simulation of ultrasound signals from flowing blood

Reading material: JAJ, ch. 4 and 6, pages 63-79 and 113-129. Self study: JAJ ch. 5.

Human circulatory system

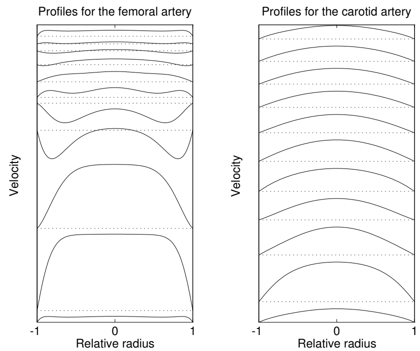


Pulmonary circulation through the lungs

Systemic circulation to the organs

Type	*Diameter [cm]*
Arteries	0.2 – 2.4
Arteriole	0.001 – 0.008
Capillaries	0.0004 – 0.0008
Veins	0.6 – 1.5

Velocity profiles for femoral and carotid artery



Profiles at time zero are shown at the bottom of the figure and time is increased toward the top. One whole cardiac cycle is covered and the dotted lines indicate zero velocity.

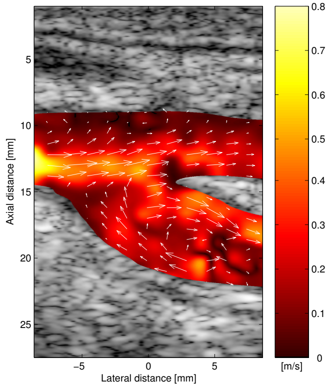
Computer simulation: flow_demo.m

Properties of blood flow in the human body

- Spatially variant
- Time variant (pulsating flow)
- Different geometric dimensions
- Vessels curves and branches repeatedly
- Can at times be turbulent
- Flow in all directions

- A velocity estimation system should be able to measure with a high resolution in time and space

- The topic of this and next lectures



Blood velocity estimation system

Determine the demands on a blood velocity estimation system based on the temporal and spatial velocity span in the human body for the carotid and femoral artery.

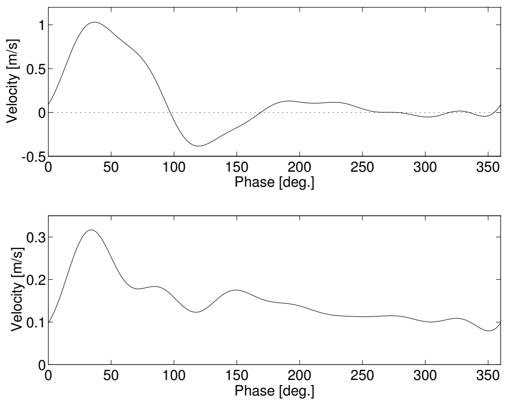
Base your assessment on slide 27 and the flow_demo.

1. What are the largest positive and negative velocities in the vessels?

2. Assume we can accept a 10% variation in velocity for one measurement. What is the longest time for obtaining one estimate?

3. What must the spatial resolution be to have 10 independent velocity estimates across the vessel?

Pulsatile flow



Spatial mean velocities from the common femoral (top) and carotid arteries (bottom).

Velocity parameters in arteries and veins

Vessel	Peak velocity cm/s	Mean velocity cm/s	Reynolds number (peak)	Pulse propagation velocity cm/s
Ascending aorta	20 − 290	10 − 40	4500	400 − 600
Descending aorta	25 − 250	10 − 40	3400	400 − 600
Abdominal aorta	50 − 60	8 − 20	1250	700 − 600
Femoral artery	100 − 120	10 − 15	1000	800 − 1030
Carotid artery	50 − 150	20 − 30		600 − 1100
Arteriole	0.5 − 1.0		0.09	
Capillary	0.02 − 0.17		0.001	
Inferior vena cava	15 − 40		700	100 − 700

Data taken from Caro et al. (1974)

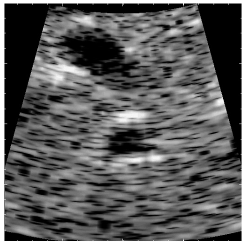
Physical dimensions of arteries and veins

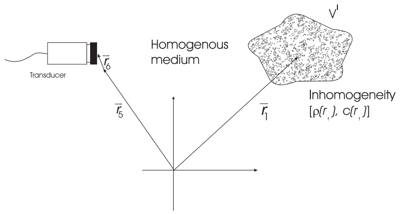
Vessel	Internal diameter cm	Wall thickness cm	Length cm	Young's modulus $N/m^2 \cdot 10^5$
Ascending aorta	1.0 − 2.4	0.05 − 0.08	5	3 − 6
Descending aorta	0.8 − 1.8	0.05 − 0.08	20	3 − 6
Abdominal aorta	0.5 − 1.2	0.04 − 0.06	15	9 − 11
Femoral artery	0.2 − 0.8	0.02 − 0.06	10	9 − 12
Carotid artery	0.2 − 0.8	0.02 − 0.04	10 −20	7 − 11
Arteriole	0.001 − 0.008	0.002	0.1 − 0.2	
Capillary	0.0004 − 0.0008	0.0001	0.02 − 0.1	
Inferior vena cava	0.6 − 1.5	0.01 − 0.02	20 − 40	0.4 − 1.0

Data taken from Caro et al. (1974)

Scattering of ultrasound by blood

Scattering of ultrasound





$$\begin{aligned} p_r(\vec{r}_5, t) &= v_{pe}(t) \underset{t}{\star} \int_{V'} \left[\frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c_0} \right] h_{pe}(\vec{r}_1, \vec{r}_5, t) d^3\vec{r}_1 \\ &= v_{pe}(t) \underset{t}{\star} f_m(\vec{r}_1) \underset{r}{\star} h_{pe}(\vec{r}_1, t) \end{aligned}$$

Electrical impulse response: $v_{pe}(t)$
Transducer spatial response: $h_{pe}(\vec{r}_1, \vec{r}_5, t) = h_t(\vec{r}_1, \vec{r}_5, t) \underset{t}{\star} h_r(\vec{r}_5, \vec{r}_1, t)$

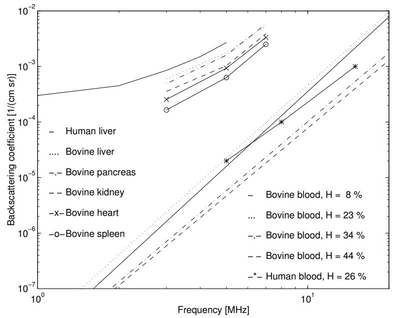
Back-scattering term: $f_m(\vec{r}_1) = \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c_0}$

Constituents of blood

	Mass density g/cm^3	Adiabatic compressibility 10^{-12} cm/dyne	Size μm	Particles per mm^3
Erythrocytes	1.092	34.1	2×7	$5 \cdot 10^6$
Leukocytes	-	-	9 - 25	$8 \cdot 10^3$
Platelets	-	-	2 - 4	$250 - 500 \cdot 10^3$
Plasma	1.021	40.9	-	-
0.9% saline	1.005	44.3	-	-

Properties of the main components of blood. Data from Carstensen et al. (1953), Dunn et al. (1969), Ulrick (1947), and Platt (1969)

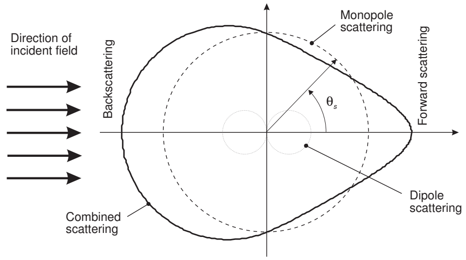
Scattering from blood I



Back-scattering from blood and different tissues

Scattering from a volume of scatterers per solid angle

Scattering from blood II



Backscattering from blood due to:

compressibility perturbations (monopole scattering)

density perturbations (dipole scattering)

Scattering cross section:

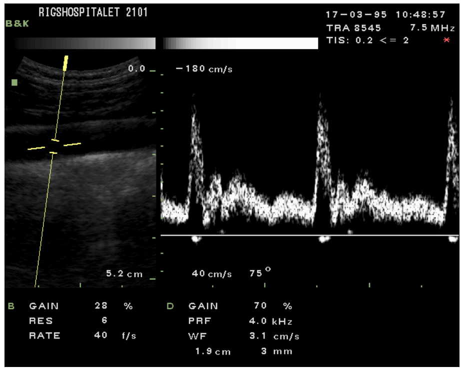
$$\sigma_d(\Theta_s) = \frac{V_e^2 \pi^2}{\lambda_0^4} \left[\frac{\kappa_e - \kappa_0}{\kappa_0} + \frac{\rho_e - \rho_0}{\rho_e} \cos \Theta_s \right]^2$$

Note that $\lambda = c/f$ so power depends on f^4 (Rayleigh scattering)

ρ_0	mean density	κ_0	mean compressibility
ρ	small perturbations in density	κ	small perturbations in compressibility
V_e	volume of the scatterer	λ_0	wavelength of incident plane, monochromatic field

Interaction between flowing blood and ultra-sound

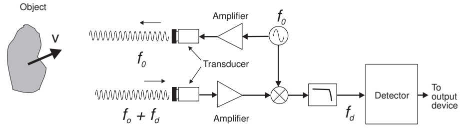
Spectral flow system



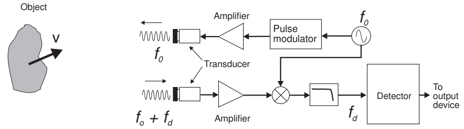
Duplex scan showing both B-mode image and spectrogram of the carotid artery. The range gate is shown as the broken line in the gray-tone image. The square brackets indicate position and size of the range gate.

The classical Doppler effect

Continuous wave system



Pulsed wave system



Doppler shift:

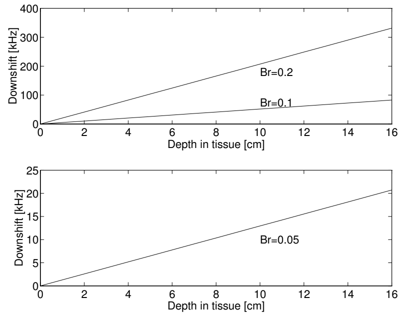
$$f_d = \frac{2v}{c} f_0$$

f_0	Center frequency of transducer
c	Speed of sound (1540 m/s)
v	Blood velocity

Typical values: $f_0 = 5$ MHz, $v = 0 - 1$ m/s.

Doppler shifts: $f_d = 0 - 6.5$ kHz.

Effect of attenuation



Down-shift in center frequency:

$$f_{mean} = f_0 - (\beta_1 B_r^2 f_0^2) z$$

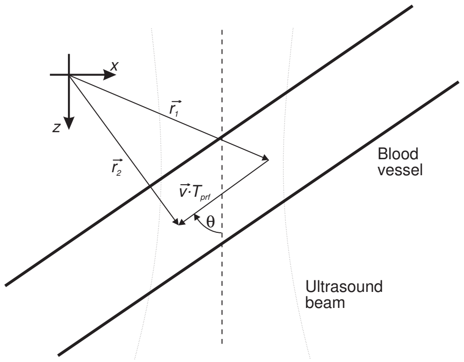
$f_0 = 3$ MHz,
$\beta_1 = 0.5$ dB/[MHz·cm]

Typical Doppler shifts are 500 to 2000 Hz!

f_0	Center frequency of transducer
c	Speed of sound
v	Blood velocity
β_1	Frequency dependent attenuation
B_r	Relative bandwidth of pulse

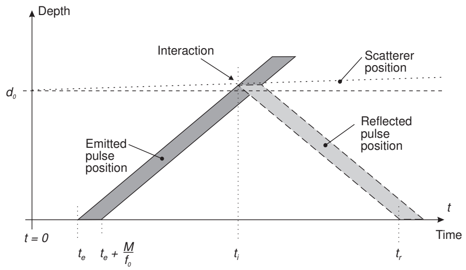
What is wrong here?

Basic measurement situation



$\vec{v}$	- Blood velocity	T_{prf}	- Time between pulse emissions
$\vec{r}_1$	- Position at first emission	$\vec{r}_2$	- Position at second emission
θ	- Angle between ultrasound beam and blood velocity		

Time-space diagram

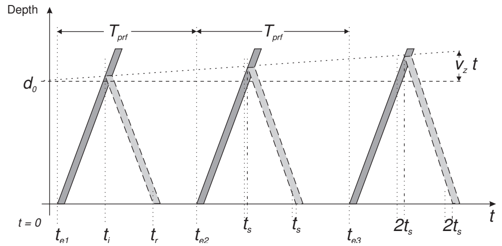


Doppler and time-shift effect for one emission:

$$\begin{aligned} r_i(t) &= g_i(t') \sin\left(2\pi f_0 \alpha \left(t - \frac{2d}{c - v_z}\right)\right) \\ \alpha &= \frac{c - v_z}{c + v_z} \\ t' &= \alpha \left(t - \frac{2d}{c - v_z}\right) \end{aligned}$$

$g_i(t)$	Envelope of emitted pulse	i	Pulse number
v_z	Blood velocity along ultrasound direction	T_{prf}	Time between pulse emissions
c	Speed of sound	d	Depth of investigation
f_0	Center frequency of transducer		

Time-space diagram for a number of pulse emissions and receptions



Time shift between emissions:

$$t_s = \frac{2|\vec{v}|\cos\theta}{c} T_{prf} = \frac{2v_z}{c} T_{prf}$$

Received signals:

$$\begin{aligned} y_1(t) &= a \cdot e(t - \frac{2d}{c}) \\ y_2(t) &= a \cdot e(t - \frac{2d}{c} - t_s) = y_1(t - t_s) \end{aligned}$$

$\vec{v}$	Blood velocity	v_z	Blood velocity along ultrasound direction
T_{prf}	Time between pulse emissions	θ	Angle between ultrasound beam and velocity
$e(t)$	Emitted signal		

Model for the received signals (single scatterer)

First emission:

$$r_0(t) = a \sin(2\pi f_0 (t_p - \frac{2d}{c}))$$

Second emission:

$$r_1(t) = a \sin(2\pi f_0 (t_p - \frac{2d}{c} - t_s))$$

i'th emission:

$$r_i(t) = a \sin(2\pi f_0 (t_p - \frac{2d}{c} - t_s i))$$

Final received signals (single scatterer)

Measurement at one fixed time t_z or depth:

$$\phi = 2\pi f_0 (t_z - \frac{2d}{c})$$

gives

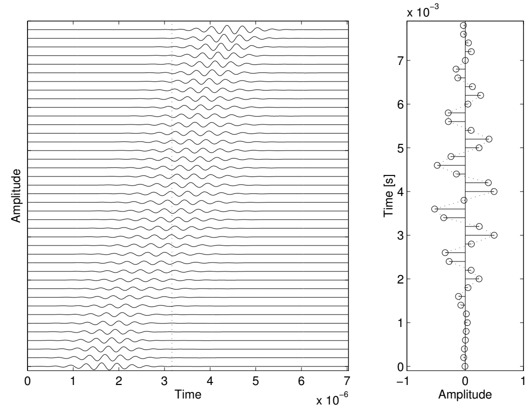
$$r_i(t_x) = -a \sin(2\pi f_0 t_s i - \phi) = -a \sin(2\pi \frac{2v_z}{c} f_0 T_{prf} i - \phi)$$

Frequency of sampled signal:

$$f_p = -\frac{2v_z}{c} f_0$$

v_z	Blood velocity along ultrasound direction	T_{prf}	Time between pulse emissions
f_0	Center frequency of transducer	c	Speed of sound
a	Scattering "strength"	t_p	Time relative to pulse emissions
t_z	Sampling time		

A simple interpretation - single scatterer

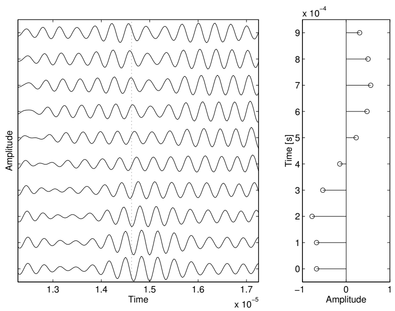


Received signal:

$$\begin{aligned} r_s(i) &= -a \sin(2\pi \frac{2v_z}{c} f_0 T_{prf} i - \phi) \\ \phi &= 2\pi f_0 \left(t_z - \frac{2d}{c}\right) \end{aligned}$$

Signal from a single moving scatterer crossing a beam from a concave transducer.

A simple interpretation - a collection of scatterers



Collection of scatterers:

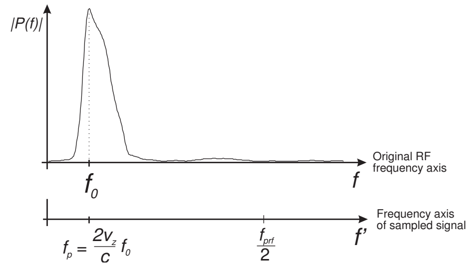
$$r_s(i) = -\sum_{k=1}^{N} a_k \sin(2\pi \frac{2v_z(k)}{c} f_0 T_{prf} i - \phi_k)$$

$$\phi_k = 2\pi f_0 \left(t_z - \frac{2d_k}{c}\right)$$

k - Scatterer number

Signal from a collection of scatterers crossing a beam from a concave transducer.

Frequency axis scaling



Spectrum of sampled signal for single scatterer moving at a velocity of v_z

Spectrum equals:

$$R(f) = P\left(\frac{c}{2v_z} f\right) * W(f)$$

$$W(f) = \frac{\sin \pi f N T_{prf}}{\sin \pi f T_{prf}} e^{-j\pi N T_{prf}}$$

Frequency scaled by:

$$\frac{2v_z}{c}$$

M	Number of cycles in pulse
$w(t)$	window due to sampling of a finite number of lines
$P(f)$	Spectrum of pulse

Physical effects

Down shift in center frequency due to attenuation:

$$\Delta f = \beta_1 B_r^2 f_0^2 d_0$$

Down shift in resulting pulsed wave spectrum:

$$\Delta f_{pw,att} = \frac{2v_z}{c} \cdot \beta_1 B_r^2 f_0^2 d_0,$$

Doppler shift due to the motion of the blood during the pulse's interaction:

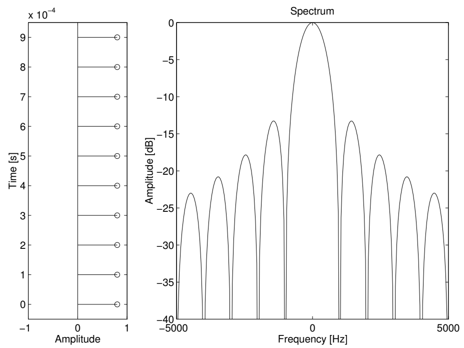
$$\Delta f_{pw,f_d} = \frac{2v_z}{c} \frac{2v_z}{c} f_0.$$

Non-linear components:

$$f_{\text{non-linear}} = \frac{2v_z}{c} f_{\text{har}}$$

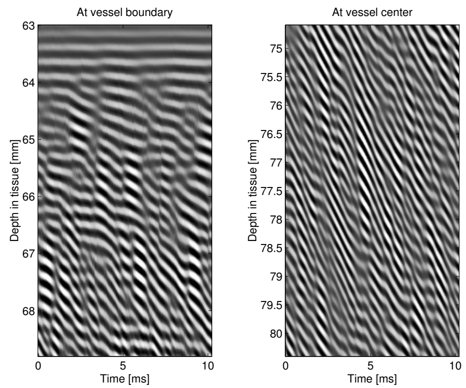
Bias depends on whether $|f_{\text{non-linear}}| > f_{prf}/2$ or not.

Spectrum for stationary signal



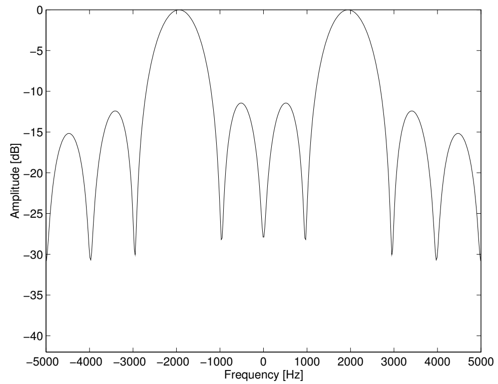
Signal obtained from stationary tissue and its spectrum.

RF signal for vessel with parabolic flow



x-direction: Time between pulse emission,
y-direction: Depth (time since one pulse emission)

Spectrum for single velocity

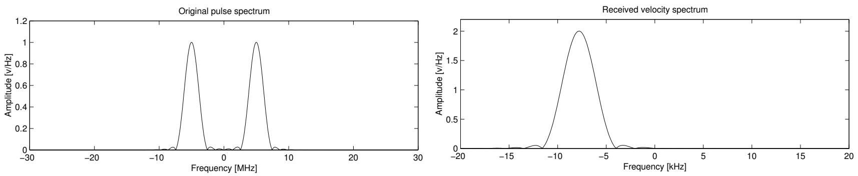


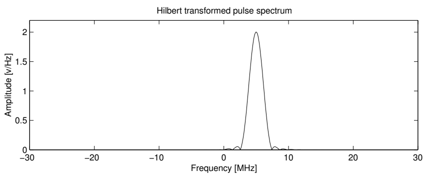
v_z = -0.5 m/s
f_0 = 3 MHz
f_{prf} = 10 kHz

Predicted frequency shift by the simple equation is:

$$-\frac{2v_z}{c}f_0 = 1948 \text{ Hz}$$

Hilbert transformation

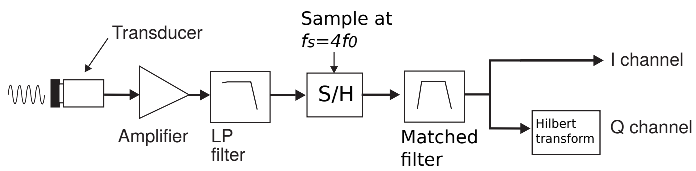




One sided spectrum created by Hilbert transforming the received signal. Thereby the sign of the frequency and velocity can be detected.

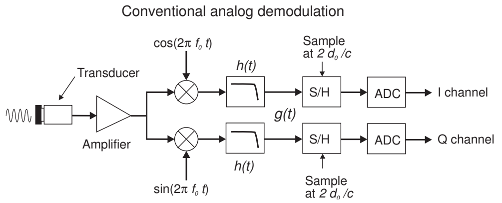
RF sampling system with Hilbert transform

RF sampling and Hilbert transform



Q channel found from Hilbert transform

Analog pulsed wave system



Demodulated signal:

$$g(t) = r(t) \cdot e^{j2\pi f_0 t} * h(t)$$

t - time since pulse emission

$$g(t) = \int_{-\infty}^{+\infty} h(\theta) r(t-\theta) e^{j2\pi f_0 (t-\theta)} d\theta = e^{j2\pi f_0 t} \int_{-\infty}^{+\infty} r(t-\theta)[e^{-j2\pi f_0 \phi} h(\theta)] d\theta$$

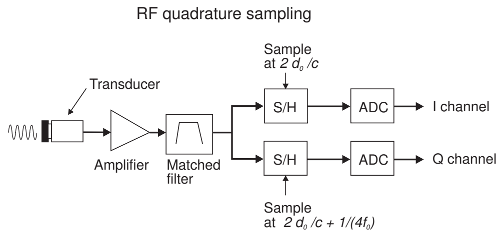
$e^{-j2\pi f_0 t} \cdot h(t)$ - Matched filter

$e^{j2\pi f_0 t}$ - Complex amplitude factor

Sampling operation: $t = t_x = \frac{2d_0}{c}$

If $t_x = \frac{K}{f_0}$ we get: $e^{j2\pi f_0 \frac{K}{f_0}} = 1$, (Note also $|e^{j2\pi f_0 \frac{K}{f_0}}| = 1$)

RF quadrature sampling system

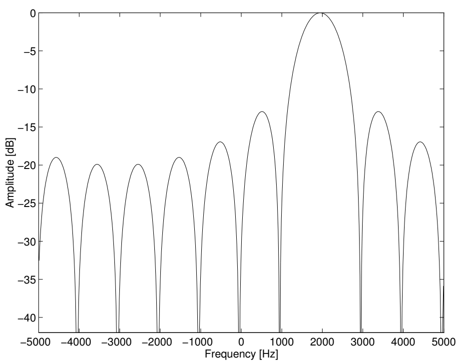


$$\sin(2\pi f_0 t) = \cos(2\pi f_0 t - \frac{\pi}{2}) = \cos(2\pi f_0 (t - \Delta\tau))$$

$$\Delta\tau = \frac{1}{4 f_0}$$

Q channel found from delayed sampling (or Hilbert transform)

Resulting spectrum of received signal after RF IQ-demodulation and sampling



$v_z = 0.5$ m/s	Blood velocity
$N = 10$	Number of acquired pulse-echo lines
$M = 7$	Number of cycles in pulse
$f_{prf} = 10$ kHz	Pulse repetition frequency

Range/velocity ambiguity

Pulse repetition frequency limited by:

$$T_{prf} = \frac{1}{f_{prf}} \geq \frac{2d_0}{c}$$

Highest velocity detectable by this system is:

$$\frac{f_{prf}}{2} \geq \frac{2v_{max}}{c} f_0.$$

Range-velocity limitation:

$$\begin{cases} f_{prf} = \frac{c}{2d_0} \\ f_{prf} = \frac{4v_{max}}{c} f_0 \end{cases}$$

$$\Downarrow$$

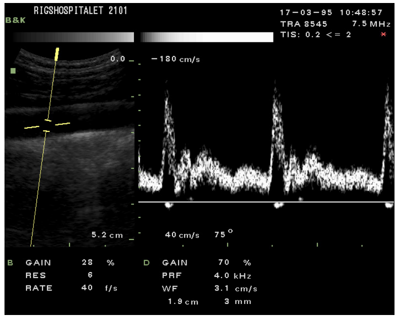
$$\frac{c}{2d_0} = \frac{4v_{max}}{c} f_0$$

Range/velocity ambiguity:

$$v_{max} = \frac{c^2}{8 d_0 f_0}$$

Calculation of the velocity spectrum

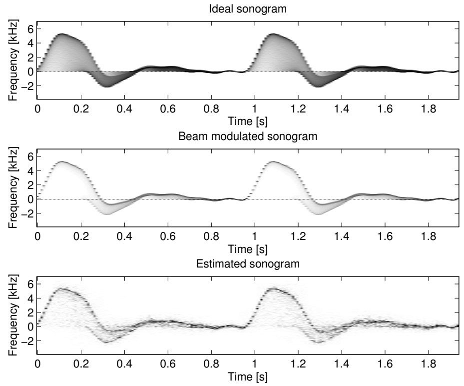
1. Sample RF signal from transducer and apply matched filter
2. Perform Hilbert transform and take out one sample per emission at range gate depth
3. Apply window on data and make a Fourier transform on the last 128 or 256 samples
4. Compress data and display for a dynamic range of 40-60 dB as a time-velocity (frequency) plot
5. Repeat this process every 1-5 ms



Spectrogram from carotid artery

This is the topic of exercise 4

Influence of beam and stochastic signal

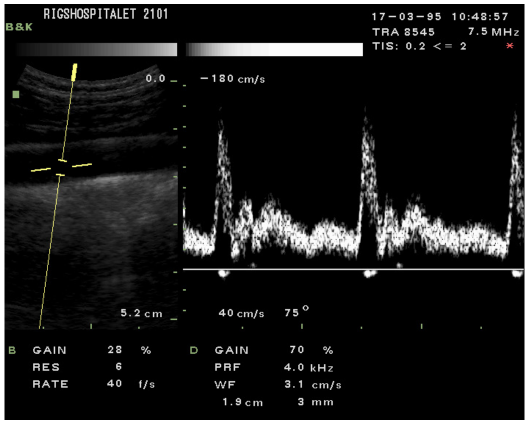


Ideal spectrogram

Central core of the vessel contributes to the spectrogram.

Effect of estimating the spectrogram from a stochastic signal.

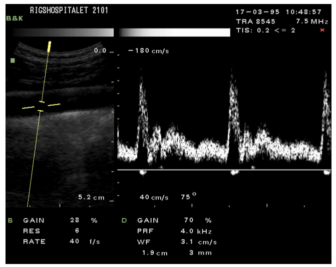
Spectrogram from carotid artery



Computer simulation: snd_demo.m

Pulse wave ultrasound systems for velocity estimation

- Weak Rayleigh scattering from blood compared to surrounding tissue
- Instantaneous Doppler shift not used, but shift in position between pulse
- Influence from different physical effects
- Description of pulsed wave system
- Finding the velocity direction
- Range/velocity ambiguity



- Can only measure the velocity distribution at one place. Would be convenient with an image of velocity
- The topic for the next lecture on Monday

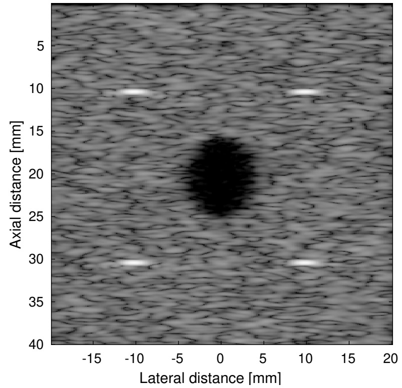
Discussion for next time

Calculate what you would get in a velocity estimation system for the time shift and the estimated frequency.

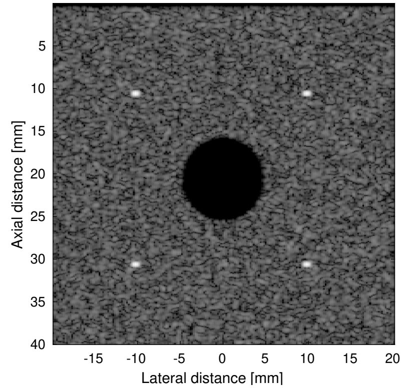
Assume a velocity of 0.75 m/s at an angle of 45 degrees. The center frequency of the probe is 3 MHz, and the pulse repetition frequency is 10 kHz. The speed of sound is 1500 m/s.

1. How much is the time shift between two ultrasound pulse emissions?
2. What would the center frequency of the received pulse wave spectrum be?
3. What is the highest velocity possible to estimate?
4. To what depth can this velocity be estimated?

Exercise 2 on ultrasound images

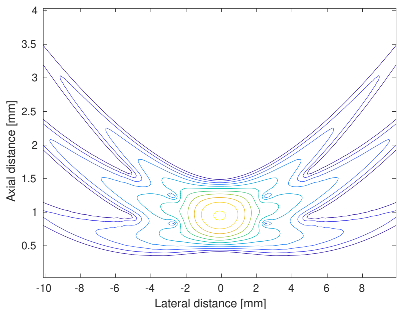


Simulated image for psf 1

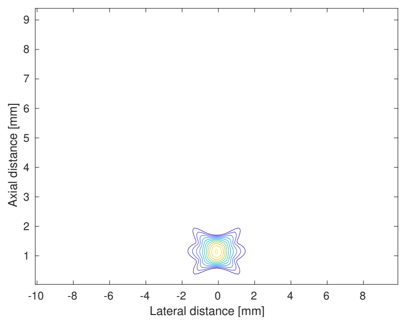


Simulated image for psf 2

Exercise 2 on ultrasound images



Point spread function 1



Point spread function 2

Exercise 3 about generating ultrasound RF flow data

Basic model, first emission:

$$r_1(t) = p(t) * s(t)$$

$s(t)$ - Scatterer amplitudes (white, random, Gaussian)

Second emission:

$$r_2(t) = p(t) * s(t - t_s) = r_1(t - t_s)$$

Time shift t_s:

$$t_s = \frac{2v_z}{c} T_{prf}$$

$r_1(t)$	Received voltage signal	$p(t)$	Ultrasound pulse
$*$	Convolution	v_z	Axial blood velocity
c	Speed of sound	T_{prf}	Time between pulse emissions

Signal processing

1. Find ultrasound pulse (load from file)
2. Make scatterers
3. Generate a number of received RF signals
4. Study the generated signals
5. Compare with simulated and measured RF data