22485 Medical Imaging systems

Lecture 4: Simulation of ultrasound signals and design of arrays

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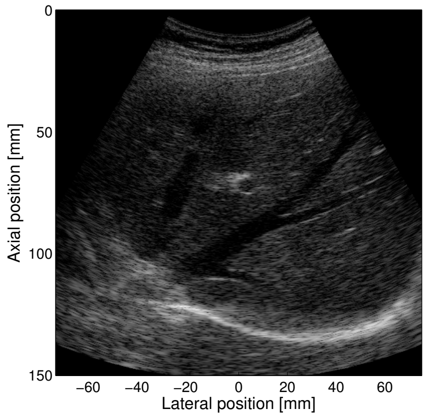
Topic of today: Ultrasound imaging with arrays and its modelling

1. Solution to exercise 1
2. Assignment from last time
3. Array imaging from last
4. Ultrasound fields and spatial impulse responses
5. Design of array geometries
6. Questions for exercise 1 and notes for exercise 2

Reading material: JAJ, ch. 2., p. 36-44

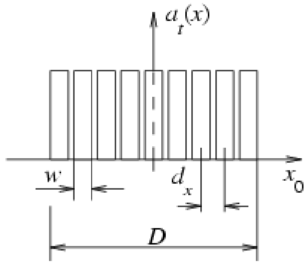
Self study: CW fields, Non-linear ultrasound will be explained in lecture 8

Solution to Exercise 1



Veins in the liver

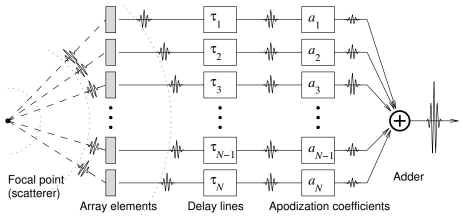
Array geometry



- d_x - Element pitch. For linear array: $\approx \lambda = c/f_0$, for phased array: $\approx \lambda/2$
- w - Width of element
- $k_e = d_x - w$ - Kerf (gap between elements)
- $D = (N_e - 1)d_x + w$ - Size of transducer

- Commercial 7 MHz linear array:
  - Elements: $N_e = 192$, 64 active at the same time
  - $\lambda = c/f_0 = 1540/7 \cdot 10^6 = 0.22$ mm
  - Pitch: $d_x = 0.208$ mm
  - Width: $D = 3.9$ cm
  - Height: $h = 4.5$ mm
  - Kerf: $k_e = 0.035$ mm

Beamforming in Modern Scanners

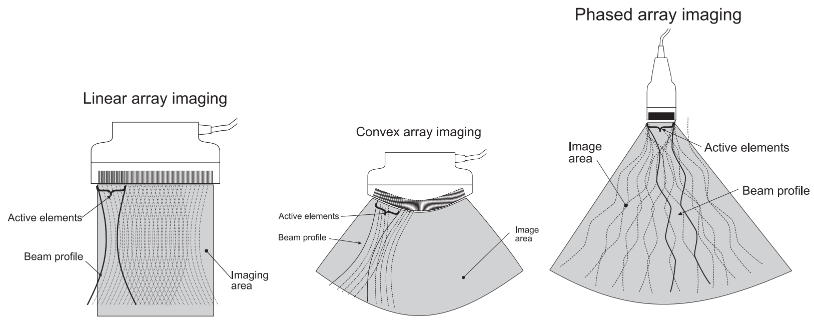


$$s(t) = \sum_{1}^{N_{xdc}} a_i y_i(t - \tau_i)$$

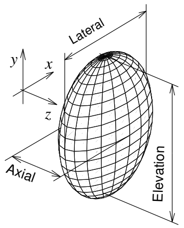
$$\tau_i = \frac{|\vec{r}_c - \vec{r}_f| - |\vec{r}_i - \vec{r}_f|}{c}$$

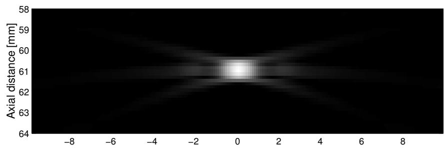
- a_i - Weighting coefficient (apodization)
- $y_i(t)$ - Received signal
- $\vec{r} = [x, y, z]^T$ - Spatial position
- $\vec{r}_i$ - Position of transducer element,
- $\vec{r}_c$ - Beam reference point
- $\vec{r}_f$ - Focal point
- c - Speed of sound

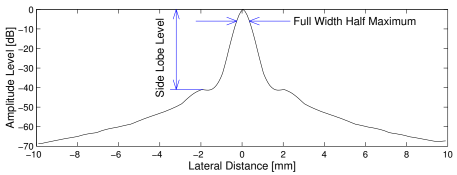
Imaging methods



PSF Characteristics







- The PSF is three dimensional
- The B-mode images are only 2-D
- Displayed on a logarithmic scale
- Maximum taken along z
- Parameters used: FWHM, side- and grating-lobe level

Discussion assignment

What are the focusing delays on an array?

Parameters: 64 element array, λ pitch, all elements used in transmit

It is a 5 MHz array, so $\lambda = 1500/5 \cdot 10^6 = 0.30$ mm

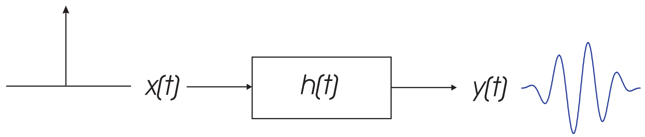
Focusing is performed directly down at the array center.

1. Imaging depth of 1 cm: How much should the center element be delayed?

2. Imaging depth of 10 cm: How much should the center element be delayed?

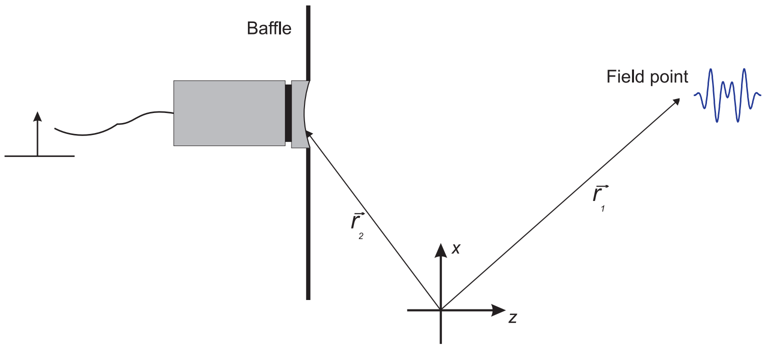
How can we calculate the ultrasound fields?

Linear Electrical System



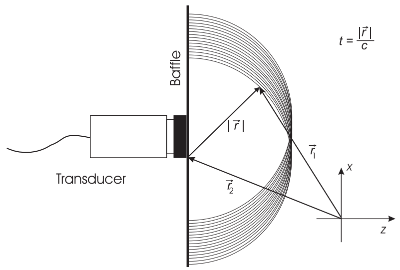
Fully characterized by it's impulse response

Linear Acoustic System



Impulse response at a point in space - Spatial Impulse Responses - $h(\vec{r}, t)$

Huygens' Principle



Arrival times: $t = d/c$, summation of spherical waves

Moving the point results in a new impulse response:

Spatial Impulse Responses - h

Rayleigh's Integral

$$
\begin{aligned}
p(\vec{r}_1, t) &= \frac{\rho_0}{2\pi} \int_S \frac{\frac{\partial v_n(\vec{r}_2, t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{\partial t}}{\mid \vec{r}_1 - \vec{r}_2 \mid} d^2\vec{r}_2 \\
&= \rho_0 \frac{\partial v_n(t)}{\partial t} \int_S \frac{\delta(t - \frac{\mid \vec{r}_1 - \vec{r}_2 \mid}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} d^2\vec{r}_2 \qquad (1)
\end{aligned}
$$

Remeber that $v_n(t) * \delta(t - t_0) = v_n(t - t_0)$

$\mid \vec{r}_1 - \vec{r}_2 \mid$	- Distance to field point
$v_n(\vec{r}_2, t)$	- Normal velocity of transducer surface. Same vibration over surface gives: $v_n(\vec{r}_2, t) = v_n(t)$

Summation of spherical waves from each point on the aperture surface

Spatial impulse response:

$$h(\vec{r}_1, t) = \int_S \frac{\delta(t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dS$$

Emitted field:

$$p(\vec{r}_1, t) = \rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1, t)$$

Pulse echo field:

$$v_r(\vec{r}_1, t) = v_{pe}(t) * h_{pe}(\vec{r}_1, t) = v_{pe}(t) * h_t(\vec{r}_1, t) * h_r(\vec{r}_1, t)$$

How do we calculate Spatial Impulse Responses?

Acoustic Reciprocity

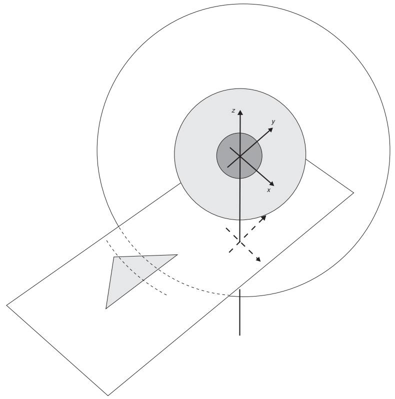
Kinsler & Frey:

"If in an unchanging environment the locations of a small source and a small receiver are interchanged, the received signal will remain the same."

In other words:

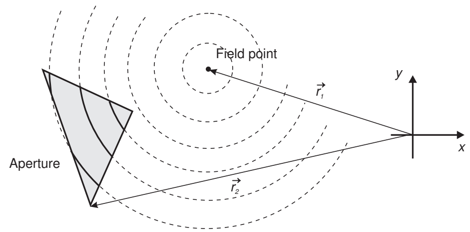
The field can be derived by emitting a spherical wave from the field point and finding the arc that intersects the aperture.

Situation



Emission of spherical wave from the field point and its intersection of the aperture.

Projection onto Aperture Plane



Intersection of spherical waves from the field point by the aperture, when the field point is projected onto the plane of the aperture.

Calculation of Spatial Impulse Responses

Spatial impulse response:

$$h(\vec{r}_1, t) = \int_S \frac{\delta(t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{2\pi|\vec{r}_1 - \vec{r}_2|} dS,$$

$\vec{r}_1$ position of field point, $\vec{r}_2$ position on aperture.

Polar coordinate system gives

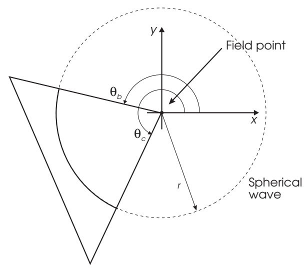
$$\int\int_s f(x,y)dxdy = \int_0^r \int_0^{2\pi} r f(r,\theta) d\theta dr.$$

Projected circles have radius: $r = \sqrt{(ct)^2 - z^2}$

Distance to field point: $R = \sqrt{z^2 + r^2}$, $\quad z$ - field point's height above $x - y$ plane.

$$h(\vec{r}_1, t) = \int_0^r \int_0^{2\pi} r \frac{\delta(t - \frac{|R|}{c})}{2\pi|R|} d\theta dr.$$

Example:



First response arrives at $t = t_1 = z/c$, hereafter the fixed part of the circle between the angles θ_b and θ_c contributes to the response.

Response is:

$$h_T(\vec{r}_1, t) = \int_0^r \int_{\theta_b}^{\theta_c} r \frac{\delta(t - \frac{|R|}{c})}{2\pi|R|} d\theta dr = \frac{\theta_c - \theta_b}{2\pi} \int_0^r r \frac{\delta(t - \frac{|R|}{c})}{|R|} dr$$

Spatial impulse response for example

Substitution for R is: $R^2 = (z^2 + r^2)$, $dR/dr = \frac{d\sqrt{z^2+r^2}}{dr} = \frac{1}{2\sqrt{z^2+r^2}} 2r = r/R \Rightarrow RdR = rdr$. Substituting this gives:

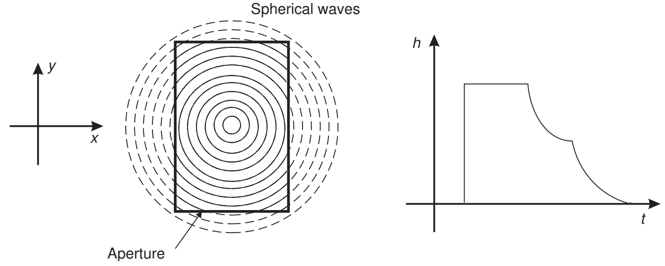
$$h_T(\vec{r}_1, t) = \frac{\theta_c - \theta_b}{2\pi} \int_z^{\sqrt{z^2+r^2}} R \frac{\delta(t - \frac{|R|}{c})}{|R|} dR = \frac{\theta_c - \theta_b}{2\pi} \int_z^{\sqrt{z^2+r^2}} \delta(t - \frac{|R|}{c}) dR$$

Time substitution $R/c = t'$ results in $(dt'/dR = 1/c, dR = cdt')$

$$\begin{aligned} h_T(\vec{r}_1, t) &= \frac{\theta_c - \theta_b}{2\pi} c \int_{z/c}^{\sqrt{z^2+r^2}/c} \delta(t - t') dt' = \frac{\theta_c - \theta_b}{2\pi} c \int_{t_1}^{t_x} \delta(t - t') dt' \\ &= \frac{(\theta_c - \theta_b)}{2\pi} c \qquad \text{for } t_1 \le t \le t_x. \end{aligned}$$

Time t_x equals the corresponding time for edge point closest to origo.

Examples of Spatial Impulse Responses



Emitted pressure field:

$$p(\vec{r}, t) = \rho_0 \frac{\partial v_n(t)}{\partial t} * h(\vec{r}, t)$$

Computer simulation: sir_demo.m

Ultrasound fields

Emitted field:

$$p(\vec{r}_1, t) = \rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1, t)$$

Pulse echo field:

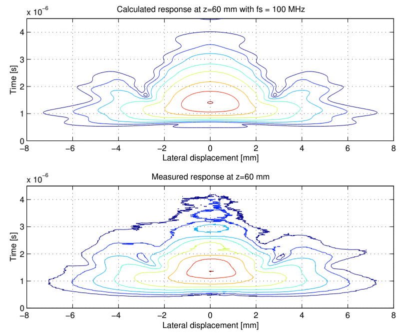
$$\begin{aligned} v_r(\vec{r}_1, t) &= v_{pe}(t) * f_m(\vec{r}_1) * h_{pe}(\vec{r}_1, t) \\ &= v_{pe}(t) * f_m(\vec{r}_1) * h_t(\vec{r}_1, t) * h_r(\vec{r}_1, t) \\ f_m(\vec{r}_1) &= \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c} \end{aligned}$$

Continuous wave fields:

$$\mathcal{F}\{p(\vec{r}_1, t)\}, \qquad \mathcal{F}\{v_r(\vec{r}_1, t)\}$$

All fields can be derived from the spatial impulse response.

Point spread functions



Point spread function for concave, focused transducer
Top: simulation top
Bottom: tank measurement (6 dB contour lines)

How do we determine the arrays geometry?

Field for arrays

Linear medium, individual spatial impulse responses are summed:

$$h_a(\vec{r}_p, t) = \sum_{i=0}^{N-1} h_e(\vec{r}_i, \vec{r}_p, t),$$

Assume elements are very small and field point is far away from the array:

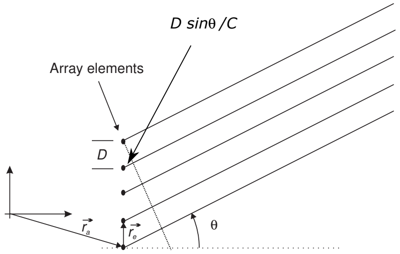
$$h_a(\vec{r}_p, t) = \frac{k_a}{R_p} \sum_{i=0}^{N-1} \delta(t - \frac{|\vec{r}_i - \vec{r}_p|}{c})$$

Note, spherical wave.

R_p - Distance to transducer

k - Constant of proportionality

Array geometry



Geometry of linear array

If spacing between elements is D, then

$$h_a(\vec{r}_p, t) = \frac{k_a}{R_p} \sum_{i=0}^{N-1} \delta\left(t - \frac{|\vec{r}_a + iD\vec{r}_e - \vec{r}_p|}{c}\right)$$

Difference in arrival time between elements far from the transducer is

$$\Delta t = \frac{D \sin \Theta}{c}.$$

Combined spatial impulse response is, thus, a series of Dirac pulses separated by Δt.

$$h_a(\vec{r}_p, t) \approx \frac{k_a}{R_p} \sum_{i=0}^{N-1} \delta\left(t - \frac{R_p}{c} - i\Delta t\right) \leftrightarrow H_a(f)$$

Usefull rules

Delay rule:

$$\delta(t - iT_0) \;\leftrightarrow\; \exp(-j2\pi f i T_0) = \exp(-j2\pi f T_0)^i$$

Power series:

$$\sum_{i=0}^{N-1} \exp\left(-j2\pi f T_0\right)^i \;=\; \frac{\sin(\pi f T_0 N)}{\sin(\pi f T_0)} \exp\left(-j2\pi f (N-1)\frac{T_0}{2}\right)$$

Beam pattern

Beam pattern as a function of angle for a particular frequency is found by Fourier transforming h_a

$$\begin{aligned} H_a(f) &= \frac{k_a}{R_p}\sum_{i=0}^{N-1} \exp\left(-j2\pi f\left(\frac{R_p}{c} + i\frac{D\sin\Theta}{c}\right)\right) \\ &= \exp(-j2\pi\frac{R_p}{c})\frac{k_a}{R_p}\sum_{i=0}^{N-1} \exp\left(-j2\pi f\frac{D\sin\Theta}{c}\right)^i \\ &= \frac{\sin(\pi f\frac{D\sin\Theta}{c}N)}{\sin(\pi f\frac{D\sin\Theta}{c})}\exp\left(-j\pi f(N-1)\frac{D\sin\Theta}{c}\right)\frac{k_a}{R_p}\exp(-j2\pi\frac{R_p}{c}). \end{aligned}$$

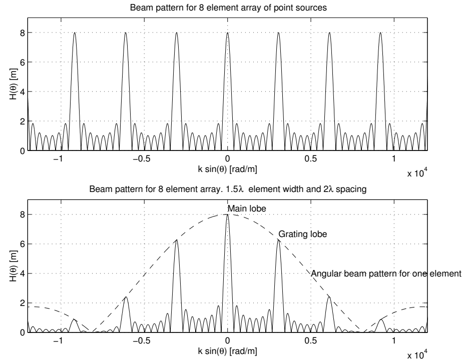
Amplitude of the beam profile:

$$|H_a(f)| = \left|\frac{k_a}{R_p}\frac{\sin(\pi N\frac{D}{\lambda}\sin\Theta)}{\sin(\pi\frac{D}{\lambda}\sin\Theta)}\right|.$$

Note correspondence to Fourier transform of digital square wave.

Continuous wave field of point sources array

$$|H_a(f)| = \left|\frac{k_a}{R_p}\frac{\sin(\pi N\frac{D}{\lambda}\sin\Theta)}{\sin(\pi\frac{D}{\lambda}\sin\Theta)}\right| = \left|A\frac{\sin(\frac{ND}{2}k\sin\Theta)}{\sin(\frac{D}{2}k\sin\Theta)}\right|, \qquad k = 2\pi/\lambda$$



Grating lobes for array with 8 point elements (top) and of 8 elements with a size of 1.5λ (bottom). The pitch is 2λ.

Interpretation and consequences

Beam profile:

$$|H_a(f)| = \left|\frac{k_a}{R_p}\frac{\sin(\pi N\frac{D}{\lambda}\sin\Theta)}{\sin(\pi\frac{D}{\lambda}\sin\Theta)}\right|$$

D - Pitch of transducer.

N - Number of elements.

ND - Width of array.

Main lobe at $\Theta = 0$ or $n = 0$. Width from zeros at:

$$N\frac{D\sin\Theta}{\lambda} = 1 \Rightarrow \Theta_w = 2\arcsin\frac{\lambda}{ND}$$

Other peaks should be avoided.

Poles in transfer function:

$$\frac{D\sin\Theta}{\lambda} = n$$

n - Integer $\neq 0$.

Corresponds to peaks in the beam pattern.

Demand for no grating lobe:

$$\frac{D\sin\Theta}{\lambda} < 1 \Rightarrow D < \frac{\lambda}{\sin\Theta}$$

For linear array: $D < \lambda$.

For phased array: $D < \lambda/2$ for safety margin for beam steering.

Note on fields

More information about ultrasound fields and their simulation can be found in:

Jørgen Arendt Jensen: *Linear description of ultrasound imaging systems*, Notes for the International Summer School on Advanced Ultrasound Imaging Technical University of Denmark June 1 to June 5, 2015.

Can be found on the web-site under Notes.

The Web-site for simulation can be found at:

http://field-ii.dk/

Discussion for next time

Design an array for cardiac imaging

Penetration depth 15 cm and 300 λ

Assume distance between ribs is maximum 3 cm

The elevation focus should be at 8 cm

1. What is the element pitch?
2. What is the maximum number of elements in the array?
3. What is the lateral resolution at 7 cm?
4. What is the F-number for the elevation focus?

Exercise 2 in generating ultrasound images

Basic model:

$$r(z,x) = p(z,x) ** s(z,x)$$

$r(z,x)$ - Received voltage signal (time converted to depth using the speed of sound)

$p(z,x)$ - 2D pulsed ultrasound field

$**$ - 2D convolution

$s(z,x)$ - Scatterer amplitudes (white, random)

z - Depth, x - Lateral distance

Signal processing

1. Find 2D ultrasound field (load from file)
2. Make scatterers with cyst hole
3. Make 2D convolution
4. Find compressed envelope data
5. Display the image
6. Compare with another pulsed field

Hint

Hint to make the scatterer map:

```
%  Make the scattere image

Nz=round(40/1000/dz);
Nx=round(40/1000/dx);
R=5/1000;
e=randn (Nz, Nx);
x=ones(Nz,1)*(-Nx/2:Nx/2-1)*dx;
z=(-Nz/2:Nz/2-1)'*ones(1,Nx)*dz;
outside = sqrt(z.^2 + x.^2) > R;
e=e.*outside;
```

Learned today

- Calculation of fields using spatial impulse response
- Influence of physical array dimensions on fields
- Remember to design the array for next time
- Prepare your code for Exercise 2

Next time: Blood flow, ch. 3 in JAJ, pages 45-61.