31545 Medical Imaging systems

Lecture 5: Blood flow in the human body

Jørgen Arendt Jensen
Department of Electrical Engineering (DTU Elektro)
Biomedical Engineering Group
Technical University of Denmark
September 17, 2018

1. Design of cardiac array
2. Basic observations about flow in the human body
   • Conservation of mass
   • Conservation of energy
   • Viscosity
   • Turbulence
3. Pulsating flow and its modeling
4. Pulse propagation and influence of geometric changes in vessels
5. Questions on Exercise 2

Reading material: JAJ, ch. 3, pages 45-61

Design an array for cardiac imaging
Penetration depth 15 cm and 300 λ
Assume distance between ribs is maximum 3 cm
The elevation focus should be at 8 cm

1. What is the element pitch?
2. What is the maximum number of elements in the array?
3. What is the lateral resolution at 7 cm?
4. What is the F-number for the elevation focus?

What are the basic properties of the blood flow in the human body?
**Human circulatory system**

- Pulmonary circulation through the lungs
- Systemic circulation to the organs

<table>
<thead>
<tr>
<th>Type</th>
<th>Diameter [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arteries</td>
<td>0.2 – 2.4</td>
</tr>
<tr>
<td>Arteriole</td>
<td>0.001 – 0.008</td>
</tr>
<tr>
<td>Capillaries</td>
<td>0.0004 – 0.0008</td>
</tr>
<tr>
<td>Veins</td>
<td>0.6 – 1.5</td>
</tr>
</tbody>
</table>

**Physical dimensions of arteries and veins**

<table>
<thead>
<tr>
<th>Vessel</th>
<th>Internal diameter cm</th>
<th>Wall thickness cm</th>
<th>Length cm</th>
<th>Young's modulus N/m² 10⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ascending aorta</td>
<td>1.4 – 2.3</td>
<td>0.08 – 0.08</td>
<td>5</td>
<td>3 – 6</td>
</tr>
<tr>
<td>Descending aorta</td>
<td>0.8 – 1.2</td>
<td>0.05 – 0.08</td>
<td>20</td>
<td>3 – 6</td>
</tr>
<tr>
<td>Abdominal aorta</td>
<td>0.5 – 1.2</td>
<td>0.04 – 0.06</td>
<td>15</td>
<td>9 – 11</td>
</tr>
<tr>
<td>Femoral artery</td>
<td>0.2 – 0.8</td>
<td>0.02 – 0.06</td>
<td>10 – 20</td>
<td>9 – 11</td>
</tr>
<tr>
<td>Carotid artery</td>
<td>0.2 – 0.8</td>
<td>0.02 – 0.04</td>
<td>10 – 20</td>
<td>7 – 11</td>
</tr>
<tr>
<td>Arteriole</td>
<td>0.001 – 0.008</td>
<td>0.002</td>
<td>0.1 – 0.2</td>
<td></td>
</tr>
<tr>
<td>Capillary</td>
<td>0.0004 – 0.0008</td>
<td>0.0001</td>
<td>0.02 – 0.1</td>
<td></td>
</tr>
<tr>
<td>Inferior vena cava</td>
<td>0.6 – 1.5</td>
<td>0.01 – 0.02</td>
<td>20 – 40</td>
<td>0.4 – 1.0</td>
</tr>
</tbody>
</table>

**Velocity parameters in arteries and veins**

<table>
<thead>
<tr>
<th>Vessel</th>
<th>Peak velocity cm/s</th>
<th>Mean velocity cm/s</th>
<th>Reynolds number (peak)</th>
<th>Pulse propagation velocity cm/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ascending aorta</td>
<td>20 – 290</td>
<td>10 – 40</td>
<td>4500</td>
<td>400 – 600</td>
</tr>
<tr>
<td>Descending aorta</td>
<td>25 – 250</td>
<td>10 – 40</td>
<td>3400</td>
<td>400 – 600</td>
</tr>
<tr>
<td>Abdominal aorta</td>
<td>50 – 60</td>
<td>8 – 20</td>
<td>1250</td>
<td>700 – 600</td>
</tr>
<tr>
<td>Femoral artery</td>
<td>100 – 120</td>
<td>10 – 15</td>
<td>1000</td>
<td>800 – 1030</td>
</tr>
<tr>
<td>Carotid artery</td>
<td>50 – 150</td>
<td>20 – 30</td>
<td>600 – 1100</td>
<td></td>
</tr>
<tr>
<td>Arteriole</td>
<td>0.5 – 1.0</td>
<td>0.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capillary</td>
<td>0.02 – 0.17</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inferior vena cava</td>
<td>15 – 40</td>
<td>700</td>
<td>100 – 700</td>
<td></td>
</tr>
</tbody>
</table>

Reynolds number:

\[ R_e = \frac{2R \rho \bar{v}}{\mu} \]

Indicates turbulence if \( R_e > 2500 \) (approximately).

\( R \) - Vessel radius, \( \rho \) - Density of blood, \( \mu \) - Viscosity, \( \bar{v} \) - Mean cross-sectional velocity.

Data taken from Caro et al. (1974)
Characteristics of blood flow in humans
• Pulsating flow, repetition from 1 to 3 beats/sec
• Not necessarily laminar flow
• Short entrance lengths
• Branching
• Reynolds numbers usually below 2500, non-turbulent flow
• Very complicated flow patterns
• We will start from simple observations and then develop the theory

Stationary flow properties
Laminar flow. Particles flow in parallel layers with streamlines that do not intersect.
No acceleration at a fixed position:
\[ \frac{\partial v}{\partial t} \bigg|_{x,y,z} = 0 \]
Note, however, that it is possible that \( \nabla v \neq 0 \) at changes in geometry

Conservation of mass
Mass at both ends must be equal:
\[ A_1 v_1 \rho_1 \Delta t = A_2 v_2 \rho_2 \Delta t, \]
Incompressible fluid: \( \rho_1 = \rho_2 \):
\[ v_2 = \frac{A_1}{A_2} v_1 \]
\( A_2 > A_1 \): Velocity decreases
\( A_2 < A_1 \): Velocity increases

Methods for generating a flow:
• Difference in pressure
• Difference in height

Benoulli’s law for conservation of energy
\[ p_1 + \frac{v_1^2}{2} \rho_1 + g h_1 \rho_1 + \frac{U'_1}{\uparrow} = p_2 + \frac{v_2^2}{2} \rho_2 + g h_2 \rho_2 + \frac{U'_2}{\uparrow} \]
Pressure, kinetic, potential, internal (heat, chemical, friction)

Assume same height \( h_1 = h_2 \), incompressible fluid \( \rho_1 = \rho_2 = \rho \), and same internal energy gives
\[ p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2) \]
Relations for a pressure difference

Equations:

\[ p_1 - p_2 = \frac{\rho}{2} (v_2^2 - v_1^2) \]

\[ v_2 = \frac{A_1}{A_2} v_1 \]

Combining gives:

\[ p_2 = p_1 - \frac{\rho}{2} \left( \left( \frac{A_1}{A_2} \right)^2 v_2^2 - v_1^2 \right) \]
\[ = p_1 + \frac{\rho}{2} \left( 1 - \left( \frac{A_1}{A_2} \right)^2 \right) v_1^2 \]

Examples:

- \( A_2 < A_1 \Rightarrow p_2 < p_1 \) and \( v_2 > v_1 \)
- \( A_2 > A_1 \Rightarrow p_2 > p_1 \) and \( v_2 < v_1 \)

(Imcompressible fluid and same height)

Viscosity properties

- Newtonian fluids have a linear relation between shear stress and \( \frac{v}{d} \).
- For non-Newtonian fluids \( \mu \) is dependent on shear stress \( S_{xy} \).
- Unit of viscosity is \( \text{kg}/[\text{m s}] \) or centipoise (cP (g/[cm s]))
- Value for water is 1 cP.
- Blood is a Newtonian fluid in major vessels
- The viscosity in blood depends on hematocrit (fraction of red blood cell volume), temperature, and pressure.
- In major vessel the viscosity is usually around 4 cP at 37°C and a hematocrit of 45%.
- Viscosity will give rise to parabolic velocity profiles for a laminar, stationary flow.

Flow models: Stationary parabolic flow

- Stationary flow: no pulsation
- Laminar flow
- Long entrance length

Profile:

\[ v(r) = \left( 1 - \frac{r^2}{R^2} \right) v_0. \]

Spatial mean velocity over cross section:

\[ \bar{v} = \frac{v_0}{2} \]

\( v_0 \) - Peak velocity at center of vessel, \( r \) - Radial position in vessel, \( R \) - Radius of vessel.
Poiseuille flow

Pressure difference for a laminar, parabolic flow:

\[ \Delta p = R_f Q \]

- \( R_f \) - Viscous resistance
- \( Q \) - Volume flow velocity

\[ Q = A \bar{v} = \int \int v(x,y) \, dx \, dy \] [m³/s]

\[ R_f = \frac{8 \mu l}{\pi R^4} \]

- \( l \) - Distance for pressure drop
- \( R \) - Radius of vessel

Example

Straight vessel with a diameter of 2 cm, \( \mu = 4 \) cP rises 20 cm straight up. Mean velocity is 30 cm/s and density is 1060 kg/m³.

Pressure difference between the ends of the tube?

Neglect viscosity, the pressure difference is then given by Bernoulli’s equation

\[ \Delta P = \frac{\mu l}{2} \left( \frac{v_2^2}{v_1^2} \right) + \rho g (h_2 - h_1) \]

Straight vessel gives \( v_1 = v_2 \):

\[ \Delta P = \rho g (h_2 - h_1) = 1060 \cdot 9.82 \cdot 0.2 = 2082 \text{ Pa} \]

Pressure drop due to viscosity is

\[ \Delta P = \frac{8 \mu l}{\pi R^4} \]

For a vessel with a diameter of 0.2 cm and a mean velocity of 10 cm/s, the pressure drop due to viscosity is

\[ \Delta P = \frac{8 \cdot 0.004 \cdot 0.2}{0.0001} = 640 \text{ Pa} \]

There is, thus, a considerable difference in the pressure relationships between a standing person and a person lying flat on a bed.

Why doesn’t the Giraffe explode?

What is the change in pressure in a Giraffe’s neck, when it moves it’s head from drinking to eating leaves in a tree?

The height of a giraffe is up to 6 m.

Image courtesy of Hans Nygaard, Aarhus University.
Turbulent flow in tubes

Photo of dye injected into a tube with flowing water. The images are taken of the original apparatus as used by Reynold in 1883.

Velocity is increasing from the top tube to the bottom one.

Courtesy of C. Lowe, Manchester School of Engineering.

Turbulent flow in tubes

Determined by Reynolds number:

\[ Re = \frac{2R\rho}{\mu \bar{v}} \]

Laminar flow usually found if:

\[ Re < 2000 \]

Turbulent flow found if:

\[ Re > 2500 \]

Usually non-turbulent flow is found in most vessels

Disturbed flow in the body

Normally non-turbulent flow exists in the body, but it can appear at constrictions, directional changes, and bifurcations:

Display of velocity direction and magnitude in the carotid bifurcation right after peak systole for a normal volunteer (Courtesy of Jesper Udesen).

Entrance effects

Development of a steady-state parabolic velocity profile at the entrance to a tube.

Entrance length:

\[ Ze \approx RRe \]

Example (aorta):

\[ Ze \approx 0.01 \cdot \frac{1600}{15} \approx 1 \text{m} \]
Theoretical velocity profiles

Profiles:
\[ v(r) = \left(1 - \frac{r}{R}\right)^{p_0} v_0 \]

Spatial mean velocity over cross section:
\[ \bar{v} = \left(1 - \frac{2}{p_0 + 2}\right) v_0 \]

\( v_0 \) - Peak velocity at center of vessel
\( r \) - Radial position in vessel
\( R \) - Radius of vessel

Profile at time zero are shown at the bottom of the figure and time is increased toward the top. One whole cardiac cycle is covered and the dotted lines indicate zero velocity.

Modeling pulsatile flow: I

Periodic waveform:
Fourier decomposition of spatial average velocity:
\[ V_m = \frac{1}{T} \int_0^T \bar{v}(t) \exp(-jm\omega t) dt. \]
The spatial average velocity is then:
\[ \bar{v}(t) = v_0 + \sum_{m=1}^\infty |V_m| \cos(m\omega t - \phi_m) \]
\[ \phi_m = \arg V_m. \]

Spatial mean velocities from the common femoral (top) and carotid arteries (bottom).

Modeling pulsatile flow: II

Newtonian fluid makes it possible to add Fourier components

Sinusoidal pressure difference:
\[ p(t) = \Delta P \cos(\omega t - \phi) \]

Volume flow using Womersley's formula:
\[ Q(t) = \frac{8 M_f}{R_f} \alpha^2 \sin(\omega t - \phi + \epsilon' \phi) \]
\[ \alpha = \frac{R_f}{\sqrt{\rho \mu}} \] - Womersley’s number
\[ R_f = \frac{8 \mu L}{\pi R^4} \]

\( \alpha < 1 \): Pressure and flow rate in phase
\( \alpha > 1 \): Pressure and flow rate out-of-phase

The Womersley’s parameters \( \alpha^2/M_{10} \) and \( \epsilon'_{10} \) as a function of \( \alpha \).
Modeling pulsatile flow: III

Find spatial and temporal distribution from Womersley-Evans theory:

\[ v(t, r/R) = 2v_0 \left(1 - \left(\frac{r}{R}\right)^2\right) + \sum_{m=1}^{\infty} |V_m| |\psi_m(r/R, \tau_m)| \cos(m\omega t - \phi_m + \chi_m(r/R, \tau_m)) \]

\[ v_m(t, r/R) = |V_m| |\psi_m(r/R, \tau_m)| \cos(\omega_m t - \phi_m + \chi_m) \]

\[ \psi_m(r/R, \tau_m) = \frac{\tau_m J_0(\tau_m) - \tau_m J_1(\tau_m)}{\tau_m J_0(\tau_m) - 2 J_1(\tau_m)} \]

\[ \chi_m(r/R, \tau_m) = \frac{\psi(r/R, \tau_m)}{\tau_m} \]

\[ \tau_m = j^{3/2} R \sqrt{\frac{\rho}{\mu \omega}} \]

\[ R \text{- Vessel radius, } r \text{- Radial position in vessel, } \rho \text{- Density of blood, } \mu \text{- Viscosity, } m \text{- Harmonic number} \]

---

Pulse propagation velocity

How fast a pressure disturbance travels in a vessel

Moens–Korteweg equation:

\[ c_p = \sqrt{\frac{E h}{2 \rho R}} \]

Assumes thin wall.

A more precise equation by Nichols and O’Rourke (1990):

\[ c_p = \sqrt{\frac{E h (1 - \sigma^2)}{2 \rho R}} \]

\[ \sigma = 0.5 \text{ decreases propagation velocity by } \sqrt{\frac{3}{4}}. \]

Normally the propagation velocity is 5 to 10 m/s.

\[ h \text{- wall thickness, } \rho \text{- density of the wall, } R \text{- vessel radius} \]

\[ E \text{- Young’s modulus for elasticity of vessel wall} \]

\[ \sigma \text{- ratio of transverse to longitudinal strain called Poisson ratio (often assumed 0.5)} \]

---

Fourier components for flow velocity

<table>
<thead>
<tr>
<th>Common femoral</th>
<th>Common carotid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>8.4 mm</td>
</tr>
<tr>
<td>Heart rate</td>
<td>62 bpm</td>
</tr>
<tr>
<td>Viscosity</td>
<td>0.004 kg/[m·s]</td>
</tr>
</tbody>
</table>

| m | f | \( |V_m|/v_0\) | \( \phi_m\) | m | f | \( |V_m|/v_0\) | \( \phi_m\) |
|---|---|----------|----------|---|---|----------|----------|
| 0 | - | 1.00     | 0        | 0 | - | 1.00     | 0        |
| 1 | 1.03 | 5.5 | 1.89 | 32 | 1 | 1.03 | 3.9 | 0.33 | 74 |
| 2 | 2.05 | 7.7 | 2.49 | 85 | 2 | 2.05 | 5.5 | 0.24 | 79 |
| 3 | 3.08 | 9.5 | 1.28 | 156 | 3 | 3.08 | 6.8 | 0.24 | 121 |
| 4 | 4.10 | 10.9 | 0.32 | 193 | 4 | 4.10 | 7.8 | 0.12 | 146 |
| 5 | 5.13 | 12.2 | 0.27 | 133 | 5 | 5.13 | 8.7 | 0.11 | 147 |
| 6 | 6.15 | 13.4 | 0.32 | 155 | 6 | 6.15 | 9.6 | 0.13 | 179 |
| 7 | 7.18 | 14.5 | 0.28 | 195 | 7 | 7.18 | 10.3 | 0.06 | 233 |
| 8 | 8.21 | 15.5 | 0.01 | 310 | 8 | 8.21 | 12.4 | 0.04 | 218 |

Data from Evans et al (1989)

---

Velocity parameters in arteries and veins

<table>
<thead>
<tr>
<th>Vessel</th>
<th>Peak velocity cm/s</th>
<th>Mean velocity (peak) cm/s</th>
<th>Reynolds number</th>
<th>Pulse propagation velocity cm/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ascending aorta</td>
<td>20 – 290</td>
<td>10 – 40</td>
<td>4500</td>
<td>400 – 600</td>
</tr>
<tr>
<td>Descending aorta</td>
<td>25 – 250</td>
<td>10 – 40</td>
<td>3400</td>
<td>400 – 600</td>
</tr>
<tr>
<td>Abdominal aorta</td>
<td>50 – 60</td>
<td>8 – 20</td>
<td>1250</td>
<td>700 – 600</td>
</tr>
<tr>
<td>Femoral artery</td>
<td>100 – 120</td>
<td>10 – 15</td>
<td>1000</td>
<td>800 – 1030</td>
</tr>
<tr>
<td>Carotid artery</td>
<td>50 – 150</td>
<td>20 – 30</td>
<td>600</td>
<td>600 – 1100</td>
</tr>
<tr>
<td>Arteriole</td>
<td>0.5 – 1.0</td>
<td>0.09</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>Capillary</td>
<td>0.02 – 0.17</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inferior vena cava</td>
<td>15 – 40</td>
<td>700</td>
<td>100 – 700</td>
<td></td>
</tr>
</tbody>
</table>

Data taken from Caro et al. (1974)

Computer simulation: flow_demo.m
**Vessel branching**

- Cross-sectional area of arterial tree is gradually increased
- The ratio between the total areas of the branching vessels in the human body is on the order of 1.26
- Increase in pressure gradient for the bifurcation is on the order of $\frac{2}{1.26^2} = 1.26$.
- Decrease in velocity is by a factor of 1.26, thus $v_2 = 0.8v_0$.

**Influence of geometric changes**

- Curving of vessel change the profile.
- A parabolic profile will be skewed out from the center of the vessel due to centrifugal forces.
- Results in a profile with larger velocities closer to the center of curvature than at the far wall.
- Vessels gradually narrow and Reynold’s number is successively decreased and the flow is stabilized.

**Properties of blood flow in the human body**

- Spatially variant
- Time variant (pulsating flow)
- Different geometric dimensions
- Vessels curves and branches repeatedly
- Can at times be turbulent
- Flow in all directions
- A velocity estimation system should be able to measure with a high resolution in time and space
- The topic for the next lectures: Chapter 4 and 6, Pages 63-79 and 113-129

**Discussion for next time**

You should determine the demands on a blood velocity estimation system based on the temporal and spatial velocity span in the human body for the carotid and femoral artery.

Base your assessment on slide 27 and the flow demo.

1. What are the largest positive and negative velocities in the vessels?
2. Assume we can accept a 10% variation in velocity for one measurement. What is the longest time for obtaining one estimate?
3. What must the spatial resolution be to have 10 independent velocity estimates across the vessel?
**Exercise 2 in generating ultrasound images**

Basic model:

\[ r(z,x) = p(z,x) \ast \ast s(z,x) \]

\( r(z,x) \) - Received voltage signal (time converted to depth using the speed of sound)

\( p(z,x) \) - 2D pulsed ultrasound field

\( \ast \ast \) - 2D convolution

\( s(z,x) \) - Scatterer amplitudes (white, random)

\( z \) - Depth, \( x \) - Lateral distance

---

**Signal processing**

1. Find 2D ultrasound field (load from file)
2. Make scatterers with cyst hole
3. Make 2D convolution
4. Find compressed envelope data
5. Display the image
6. Compare with another pulsed field

---

**Hint**

Hint to make the scatterer map:

```matlab
% Make the scattere image

Nz=round(40/1000/dz);
Nx=round(40/1000/dx);
Nr=round(5/1000/dx);
e=randn(Nz, Nx);
x=ones(Nz,1)*(-Nx/2:Nx/2-1);
z=(-Nz/2:Nz/2-1)'*ones(1,Nx);
outside = sqrt(z.^2 + x.^2) > Nr*ones(Nz, Nx);
e=e.*outside;
```