31545 Medical Imaging systems

Lecture 5: Blood flow in the human body

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Topic of today: Blood flow

1. Exercise 1 solution
2. Design of cardiac array
3. Basic observations about flow in the human body
   - Conservation of mass
   - Conservation of energy
   - Viscosity
   - Turbulence
4. Pulsating flow and its modeling
5. Pulse propagation and influence of geometric changes in vessels
6. Questions on Exercise 2

Reading material: JAJ, ch. 3, pages 45-61

Signal processing in ultrasound system - Exercise 1

Resulting image

Ultrasound image of portal veins in the liver.

Useful Matlab commands

Loading of files:

```
cmd=['load in_vivo_data/8820e_B_mode_invivo_frame_no_',num2str(j)]
eval(cmd)
```

Making a movie:

```
for j=1:66

  image(randn(20))
  colormap(gray(256))
  axis image

  F(j)=getframe;
  end

%  Play the movie 5 times at 22 fr/s

movie(F,5, 22)
```

Final video in: /home/jaje/undervisning/k_22485_31545_billeder/exercises/
exercise1/solution/for_2022_32_bits/in_vivo_liver_video.mp4

Design an array for cardiac imaging

Penetration depth 15 cm and 300 λ

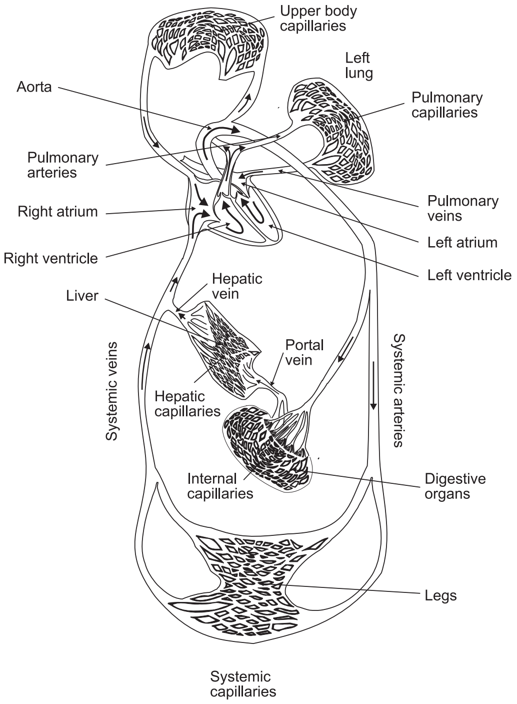
Assume distance between ribs is maximum 3 cm

The elevation focus should be at 8 cm

1. What is the element pitch?
2. What is the maximum number of elements in the array?
3. What is the lateral resolution at 7 cm?
4. What is the F-number for the elevation focus?

What are the basic properties of the blood flow in the human body?

Human circulatory system

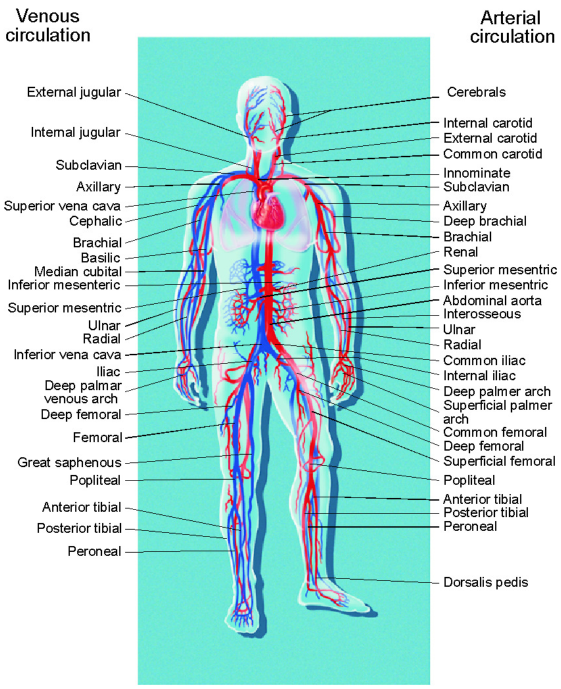


Pulmonary circulation through the lungs

Systemic circulation to the organs

Type	*Diameter [cm]*
Arteries	0.2 – 2.4
Arteriole	0.001 – 0.008
Capillaries	0.0004 – 0.0008
Veins	0.6 – 1.5

Arteries and veins in the body



Physical dimensions of arteries and veins

Vessel	Internal diameter cm	Wall thickness cm	Length cm	Young's modulus $N/m^2 \cdot 10^5$
Ascending aorta	1.0 − 2.4	0.05 − 0.08	5	3 − 6
Descending aorta	0.8 − 1.8	0.05 − 0.08	20	3 − 6
Abdominal aorta	0.5 − 1.2	0.04 − 0.06	15	9 − 11
Femoral artery	0.2 − 0.8	0.02 − 0.06	10	9 − 12
Carotid artery	0.2 − 0.8	0.02 − 0.04	10 −20	7 − 11
Arteriole	0.001 − 0.008	0.002	0.1 − 0.2	
Capillary	0.0004 − 0.0008	0.0001	0.02 − 0.1	
Inferior vena cava	0.6 − 1.5	0.01 − 0.02	20 − 40	0.4 − 1.0

Data taken from Caro et al. (1974)

Velocity parameters in arteries and veins

Vessel	Peak velocity cm/s	Mean velocity cm/s	Reynolds number (peak)	Pulse propagation velocity cm/s
Ascending aorta	20 − 290	10 − 40	4500	400 − 600
Descending aorta	25 − 250	10 − 40	3400	400 − 600
Abdominal aorta	50 − 60	8 − 20	1250	700 − 600
Femoral artery	100 − 120	10 − 15	1000	800 − 1030
Carotid artery	50 − 150	20 − 30		600 − 1100
Arteriole	0.5 − 1.0		0.09	
Capillary	0.02 − 0.17		0.001	
Inferior vena cava	15 − 40		700	100 − 700

Data taken from Caro et al. (1974)

Reynolds number:

$$R_e = \frac{2R\rho}{\mu}\bar{v}$$

Indicates turbulence if $R_e > 2500$ (approximately).

R - Vessel radius, ρ - Density of blood, μ - Viscosity, $\bar{v}$ - Mean cross-sectional velocity

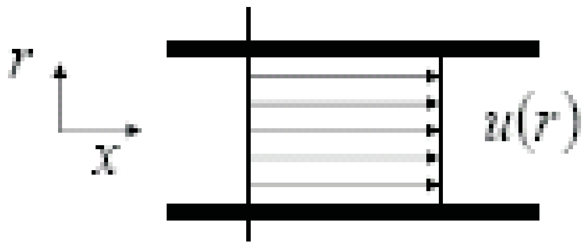
Characteristics of blood flow in humans

- Pulsating flow, repetition from 1 to 3 beats/sec
- Not necessarily laminar flow
- Short entrance lengths
- Branching
- Reynolds numbers usually below 2500, non-turbulent flow
- Very complicated flow patterns

- We will start from simple observations and then develop the theory

Stationary flow properties

Laminar flow. Particles flow in parallel layers with streamlines that do not intersect.

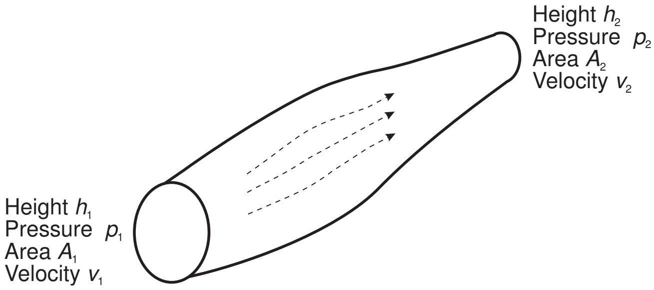


No acceleration at a fixed position:

$$\left.\frac{\partial v}{\partial t}\right|_{x,y,z} = 0$$

Note, however, that it is possible that $\nabla v \neq 0$ at changes in geometry

Conservation of mass



Mass at both ends must be equal:

$$A_1 v_1 \rho_1 \Delta t = A_2 v_2 \rho_2 \Delta t,$$

Incompressible fluid: $\rho_1 = \rho_2$:

$$v_2 = \frac{A_1}{A_2} v_1$$

$A_2 > A_1$: Velocity decreases
$A_2 < A_1$: Velocity increases

Methods for generating a flow:

- Difference in pressure
- Difference in height

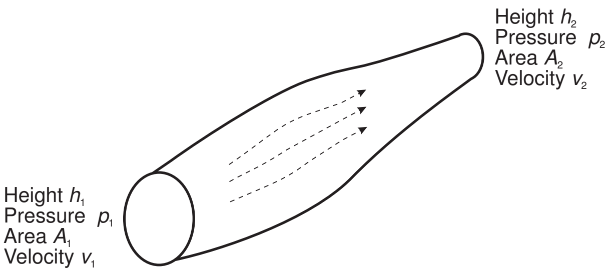
Benoulli's law for conservation of energy

$$\underset{\substack{\uparrow \\ \text{pressure}}}{p_1} + \underset{\substack{\uparrow \\ \text{kinetic}}}{\frac{v_1^2}{2}\rho_1} + \underset{\substack{\uparrow \\ \text{potential}}}{g h_1 \rho_1} + \underset{\substack{\uparrow \\ \text{Internal (heat, chemical, friction)}}}{U_1'} = p_2 + \frac{v_2^2}{2}\rho_2 + g h_2 \rho_2 + U_2'$$

Assume same height $h_1 = h_2$, incompressible fluid $\rho_1 = \rho_2 = \rho$, and same internal energy gives

$$p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2)$$

Relations for a pressure difference



Equations:

$$p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2)$$

$$v_2 = \frac{A_1}{A_2} v_1$$

Combining gives:

$$\begin{aligned} p_2 &= p_1 - \frac{\rho}{2}\left(\left(\frac{A_1}{A_2}\right)^2 v_1^2 - v_1^2\right) \\ &= p_1 + \frac{\rho}{2}\left(1 - \left(\frac{A_1}{A_2}\right)^2\right) v_1^2 \end{aligned}$$

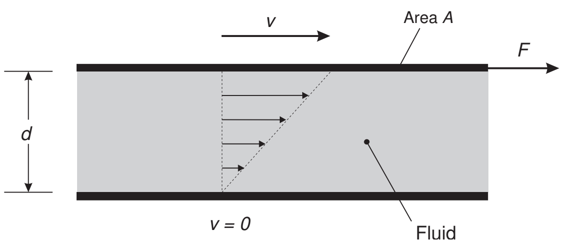
Examples:

$A_2 < A_1 \Rightarrow p_2 < p_1$ and $v_2 > v_1$

$A_2 > A_1 \Rightarrow p_2 > p_1$ and $v_2 < v_1$

(Incompressible fluid and same height)

Viscosity

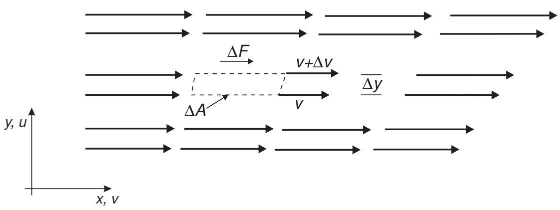


Shear stress:

$$\frac{F}{A} = \mu \frac{v_0}{d}$$

μ - Viscosity of fluid

For small fluid element:



Shear stress:

$$\frac{\Delta F}{\Delta A} = \mu \frac{\Delta v}{\Delta y}$$

In the limit:

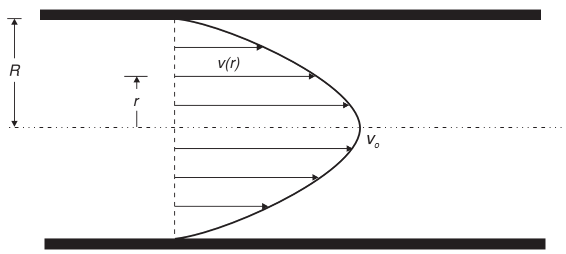
$$S_{xy} = \mu \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right)$$

for a Newtonian fluid.

Viscosity properties

- Newtonian fluids have a linear relation between shear stress and v/d.
- For non-Newtonian fluids μ is dependent on shear stress S_{xy}.
- Unit of viscosity is kg/[m s] or centipoise cP (g/[cm s])
- Value for water is 1 cP.
- Blood is a Newtonian fluid in major vessels
- The viscosity in blood depends on hematocrit (fraction of red blood cell volume), temperature, and pressure.
- In major vessel the viscosity is usually around 4 cP at 37^o and a hematocrit of 45%.
- Viscosity will give rise to parabolic velocity profiles for a laminar, stationary flow.

Flow models: Stationary parabolic flow



- Stationary flow: no pulsation
- Laminar flow
- Long entrance length

Profile:

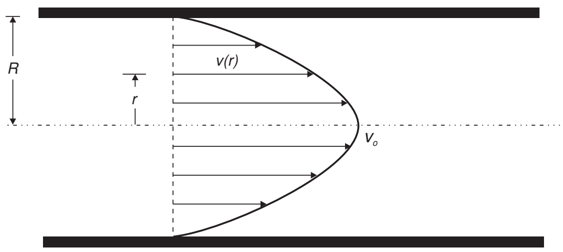
$$v(r) = \left(1 - \frac{r^2}{R^2}\right) v_0,$$

Spatial mean velocity over cross section:

$$\bar{v} = \frac{v_0}{2}$$

v_0 - Peak velocity at center of vessel, r - Radial position in vessel,
R - Radius of vessel

Poiseuille flow



Pressure difference for a laminar, parabolic flow:

$$\Delta p = R_f Q$$

R_f - Viscose resistance
Q - Volume flow velocity

$$Q = A\bar{v} = \int\int_{xy} v(x,y)dxdy \qquad [\text{m}^3/\text{s}]$$

$$R_f = \frac{8\mu l}{\pi R^4}$$

l - Distance for pressure drop
R - Radius of vessel

Example

Straight vessel with a diameter of 2 cm, $\mu = 4$ cP rises 20 cm straight up. Mean velocity is 30 cm/s and density is 1060 kg/m^3.

Pressure difference between the ends of the tube?

Neglect viscosity, the pressure difference is then given by Bernoulli's equation

$$\Delta P = \frac{\rho}{2}(v_2^2 - v_1^2) + \rho g(h_2 - h_1).$$

Straight vessel gives $v_1 = v_2$:

$$\Delta P = \rho g(h_2 - h_1) = 1060 \cdot 9.82 \cdot 0.2 = 2082 \text{ Pa}.$$

Pressure drop due to viscosity is

$$\Delta P = R_f Q = \frac{8\mu l}{\pi R^4}\bar{v}\pi R^2 = \frac{8\mu l}{R^2}\bar{v} = \frac{8 \cdot 0.004 \cdot 0.2}{0.01^2}0.3 = 19.2 \text{ Pa}.$$

For a vessel with a diameter of 0.2 cm and a mean velocity of 10 cm/s, the pressure drop due to viscosity is

$$\Delta P = \frac{8 \cdot 0.004 \cdot 0.2}{0.001^2}0.1 = 640 \text{ Pa}.$$

There is, thus, a considerable difference in the pressure relationships between a standing person and a person lying flat on a bed.

Giraffe assignment

What is the change in pressure in a Giraffe's neck, when it moves it's head from drinking to eating leaves in a tree?

The height of a giraffe is up to 6 m.

Why doesn't the Giraffe explode?

Image courtesy of Hans Nygaard, Aarhus University.

Turbulent flow in tubes

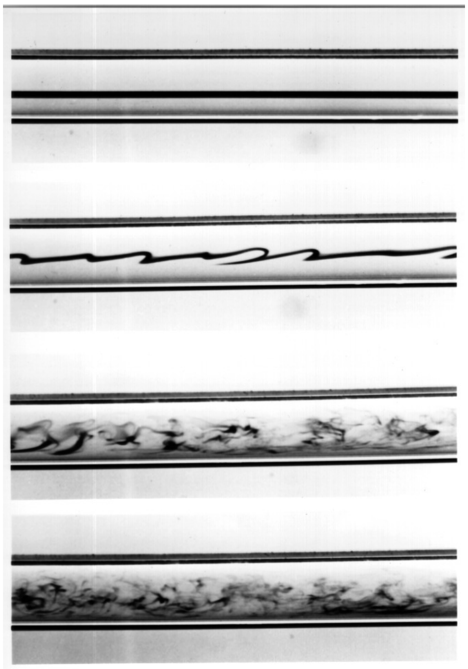


Photo of dye injected into a tube with flowing water. The images are taken of the original apparatus as used by Reynold in 1883.

Velocity is increasing from the top tube to the bottom one.

Courtesy of C. Lowe, Manchester School of Engineering.

Turbulent flow in tubes

Determined by Reynolds number:

$$R_e = \frac{2R\rho}{\mu}\bar{v}$$

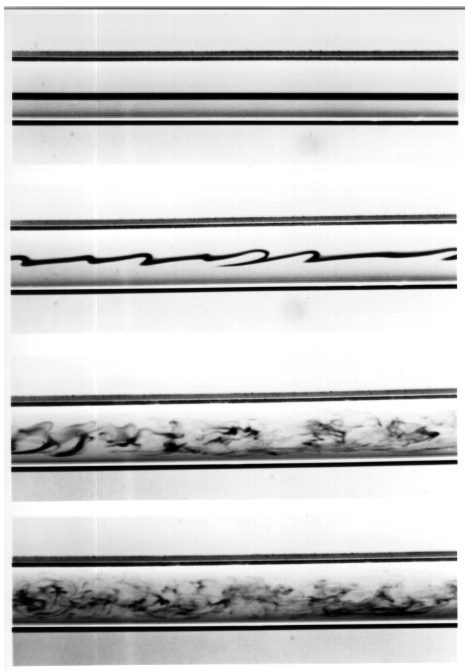
Laminar flow usually found if:

$$R_e < 2000$$

Turbulent flow found if :

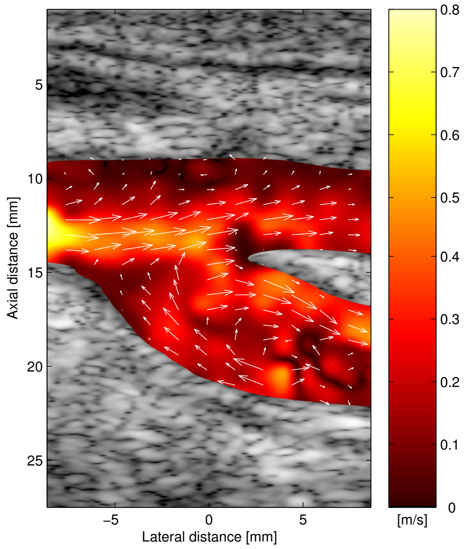
$$R_e > 2500$$

Usually non-turbulent flow is found in most vessels



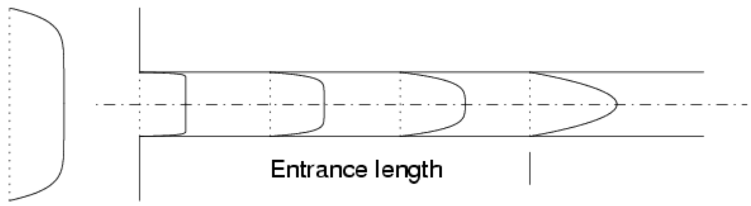
Disturbed flow in the body

Normally non-turbulent flow exists in the body, but it can appear at constrictions, directional changes, and bifurcations:



Display of velocity direction and magnitude in the carotid bifurcation right after peak systole for a normal volunteer (Courtesy of Jesper Udesen).

Entrance effects



Development of a steady-state parabolic velocity profile at the entrance to a tube.

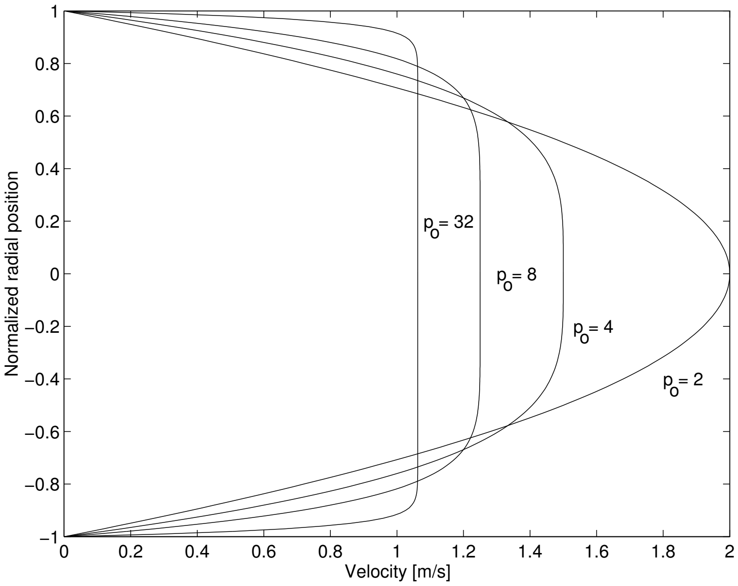
Entrance length:

$$Z_e \approx \frac{RR_e}{15}$$

Example (aorta):

$$Z_e \approx \frac{0.01 \cdot 1600}{15} \approx 1\text{m}$$

Theoretical velocity profiles



Velocity profiles for different values of p_0.

The spatial mean velocity is the same for all profiles.

Profiles:

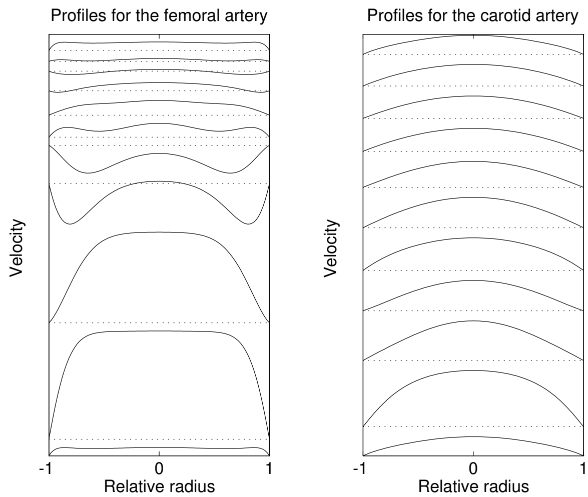
$$v(r) = \left(1 - \left(\frac{r}{R}\right)^{p_0}\right) v_0$$

Spatial mean velocity over cross section:

$$\bar{v} = v_0 \left(1 - \frac{2}{p_0 + 2}\right)$$

v_0 - Peak velocity at center of vessel
r - Radial position in vessel
R - Radius of vessel

Velocity profiles for femoral and carotid artery



Profiles at time zero are shown at the bottom of the figure and time is increased toward the top. One whole cardiac cycle is covered and the dotted lines indicate zero velocity.

Modeling pulsatile flow: I

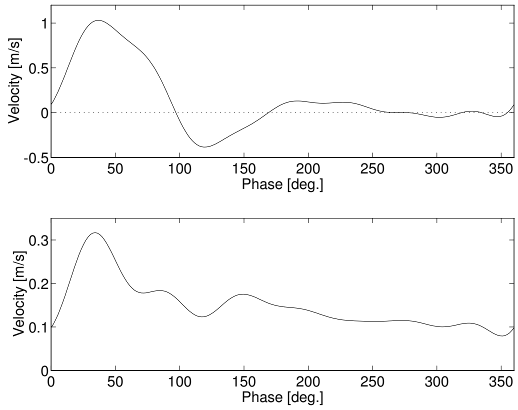
Periodic waveform:

Fourier decomposition of spatial average velocity:

$$V_m = \frac{1}{T}\int_0^T \bar{v}(t)\exp(-jm\omega t)dt.$$

The spatial average velocity is then:

$$\begin{aligned}\bar{v}(t) &= v_0 + \sum_{m=1}^{\infty} |V_m| \cos(m\omega t - \phi_m) \\ \phi_m &= \angle V_m.\end{aligned}$$



Spatial mean velocities from the common femoral (top) and carotid arteries (bottom).

Modeling pulsatile flow: II

Newtonian fluid makes it possible to add Fourier components

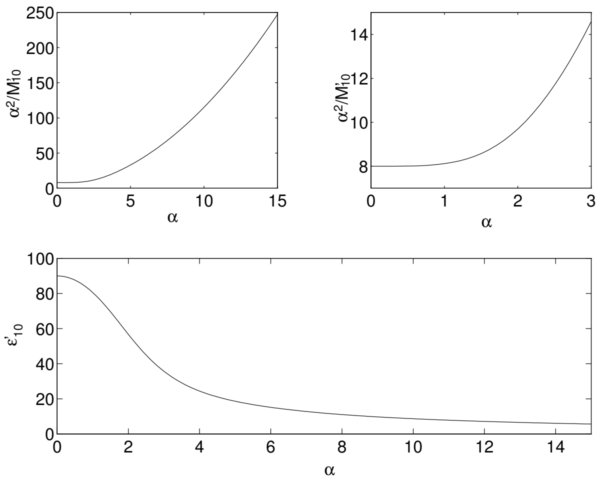
Sinusoidal pressure difference:

$$p(t) = \Delta p \cos(\omega t - \phi)$$

Volume flow using Womersley's formula:

$$\begin{aligned}Q(t) &= \frac{8}{R_f}\frac{M'_{10}}{\alpha^2}\Delta P \sin(\omega t - \phi + \epsilon'_{10}) \\ \alpha &= R\sqrt{\frac{\rho}{\mu}\omega} \quad \text{- Womersley's number} \\ R_f &= \frac{8\mu L}{\pi R^4}\end{aligned}$$

$\alpha < 1$: Pressure and flow rate in phase
$\alpha > 1$: Pressure and flow rate out-of phase



The Womersley's parameters α^2/M'_{10} and ϵ'_{10} as a function of α.

Modeling pulsatile flow: III

Find spatial and temporal distribution from Womersley-Evans theory:

$$v(t, r/R) = 2v_0\left(1 - \left(\frac{r}{R}\right)^2\right) + \sum_{m=1}^{\infty} |V_m||\psi_m(r/R, \tau_m)| \cos(m\omega t - \phi_m + \chi_m(r/R, \tau_m))$$

$$\begin{aligned}
v_m(t, r/R) &= |V_m||\psi_m(r/R, \tau_m)| \cos(\omega_m t - \phi_m + \chi_m) \\
\psi_m(r/R, \tau_m) &= \frac{\tau_m J_0(\tau_m) - \tau_m J_0(\frac{r}{R}\tau_m)}{\tau_m J_0(\tau_m) - 2J_1(\tau_m)} \\
\chi_m(r/R, \tau_m) &= \angle\psi(r/R, \tau_m) \\
\tau_m &= j^{3/2} R \sqrt{\frac{\rho}{\mu}\omega_m},
\end{aligned}$$

R - Vessel radius, r - Radial position in vessel, ρ - Density of blood,
μ - Viscosity, m - Harmonic number

Fourier components for flow velocity

Common femoral					Common carotid				
Diameter = 8.4 mm					Diameter = 6.0 mm				
Heart rate = 62 bpm					Heart rate = 62 bpm				
Viscosity = 0.004 kg/[m·s]					Viscosity = 0.004 kg/[m·s]				
m	f	α	$\|V_m\|/v_0$	ϕ_m	m	f	α	$\|V_m\|/v_0$	ϕ_m
0	-	-	1.00	-	0	-	-	1.00	-
1	1.03	5.5	1.89	32	1	1.03	3.9	0.33	74
2	2.05	7.7	2.49	85	2	2.05	5.5	0.24	79
3	3.08	9.5	1.28	156	3	3.08	6.8	0.24	121
4	4.10	10.9	0.32	193	4	4.10	7.8	0.12	146
5	5.13	12.2	0.27	133	5	5.13	8.7	0.11	147
6	6.15	13.4	0.32	155	6	6.15	9.6	0.13	179
7	7.18	14.5	0.28	195	7	7.18	10.3	0.06	233
8	8.21	15.5	0.01	310	8	8.21	12.4	0.04	218

Data from Evans et al (1989)

Computer simulation: flow_demo.m

Pulse propagation velocity

How fast a pressure disturbance travels in a vessel

Moens–Korteweg equation:

$$c_p = \sqrt{\frac{Eh}{2\rho R}},$$

Assumes thin wall.

A more precise equation by Nichols and O'Rourke (1990):

$$c_p = \sqrt{\frac{Eh}{2\rho R}(1-\sigma^2)},$$

$\sigma = 0.5$ decreases propagation velocity by $\sqrt{3/4}$.

Normally the propagation velocity is 5 to 10 m/s.

h - wall thickness, ρ - density of the wall, R - vessel radius
E - Young's modulus for elasticity of vessel wall
σ - ratio of transverse to longitudinal strain called Poisson ratio (often assumed 0.5)

Velocity parameters in arteries and veins

Vessel	Peak velocity cm/s	Mean velocity cm/s	Reynolds number (peak)	Pulse propagation velocity cm/s
Ascending aorta	20 − 290	10 − 40	4500	400 − 600
Descending aorta	25 − 250	10 − 40	3400	400 − 600
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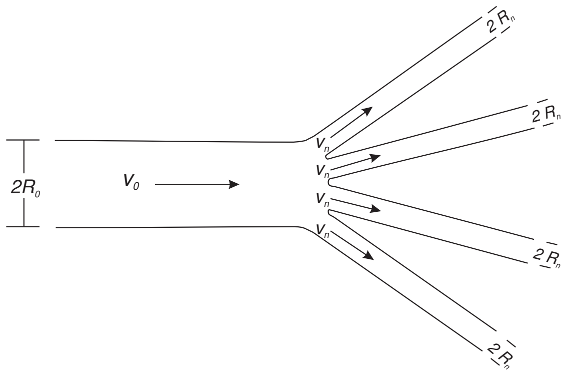
Arteriole	0.5 − 1.0		0.09	
Capillary	0.02 − 0.17		0.001	
Inferior vena cava	15 − 40		700	100 − 700

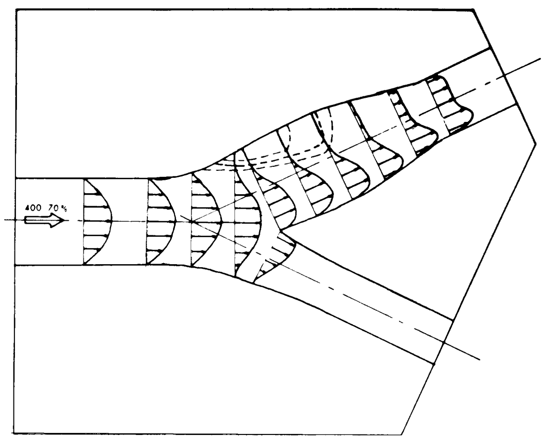
Data taken from Caro et al. (1974)

Vessel branching



- Cross-sectional area of arterial tree is gradually increased
- The ratio between the total areas of the branching vessels in the human body is on the order of 1.26
- Increase in pressure gradient for the bifurcation is on the order of $2/1.26^2 = 1.26$.
- Decrease in velocity is by a factor of 1.26, thus $v_2 = 0.8v_0$.

Influence of geometric changes



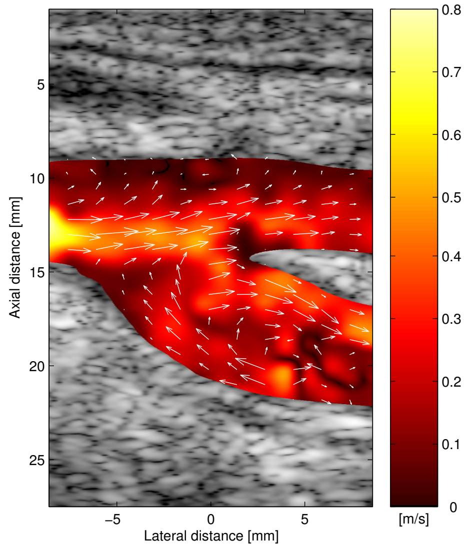
- Curving of vessel change the profile.
- A parabolic profile will be skewed out from the center of the vessel due to centrifugal forces.
- Results in a profile with larger velocities closer to the center of curvature than at the far wall.
- Vessels gradually narrow and Reynold's number is successively decreased and the flow is stabilized.

Properties of blood flow in the human body

- Spatially variant
- Time variant (pulsating flow)
- Different geometric dimensions
- Vessels curves and branches repeatedly
- Can at times be turbulent
- Flow in all directions

- A velocity estimation system should be able to measure with a high resolution in time and space

- The topic for the next lectures: Chapter 4 and 6, Pages 63-79 and 113-129



Discussion for next time

You should determine the demands on a blood velocity estimation system based on the temporal and spatial velocity span in the human body for the carotid and femoral artery.

Base your assessment on slide 29 and the flow_demo.

1. What are the largest positive and negative velocities in the vessels?

2. Assume we can accept a 10% variation in velocity for one measurement. What is the longest time for obtaining one estimate?

3. What must the spatial resolution be to have 10 independent velocity estimates across the vessel?

Exercise 2 in generating ultrasound images

Basic model:

$$r(z,x) = p(z,x) * *s(z,x)$$

$r(z,x)$ - Received voltage signal (time converted to depth using the speed of sound)

$p(z,x)$ - 2D pulsed ultrasound field

$**$ - 2D convolution

$s(z,x)$ - Scatterer amplitudes (white, random)

z - Depth, x - Lateral distance

Signal processing

1. Find 2D ultrasound field (load from file)
2. Make scatterers with cyst hole
3. Make 2D convolution
4. Find compressed envelope data
5. Display the image
6. Compare with another pulsed field

Hint

Hint to make the scatterer map:

```
% Make the scattere image

Nz=round(40/1000/dz);
Nx=round(40/1000/dx);
R=5/1000;
e=randn (Nz, Nx);
x=ones(Nz,1)*(-Nx/2:Nx/2-1)*dx;
z=(-Nz/2:Nz/2-1)'*ones(1,Nx)*dz;
outside = sqrt(z.^2 + x.^2) > R;
e=e.*outside;
```