22485 Medical Imaging systems

Lecture 3: Ultrasound focusing and modeling

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Topic of today: Imaging in ultrasound

Focusing and spatial impulse responses

1. Repetition and assignment from last
2. Arrays and focusing using delay-and-sum
3. Ultrasound fields and resolution

Reading material: JAJ, ch. 2., p. 24-36

Self-study: CW fields

From last lecture: The wave equation

Speed of sound c:

$$c = \sqrt{\frac{1}{\rho\kappa}}$$

Linear wave equation:

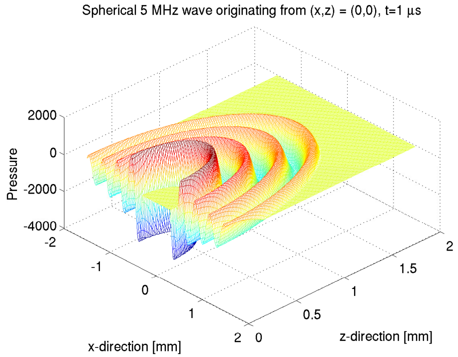
$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

General solutions:

$$p(t,x) = g(t \pm \frac{x}{c})$$

$\frac{x}{c}$ is time delay

Spherical wave



$$p(t,\vec{r}) = \frac{p_0}{|\vec{r}|}\sin(2\pi f_0(t - \frac{|\vec{r}|}{c}))$$

Propagates radially from a center point.

Used for describing diffraction, focusing, and ultrasound fields.

Attenuation of ultrasound

Tissue	Attenuation dB/[MHz·cm]
Liver	0.6 − 0.9
Kidney	0.8 − 1.0
Spleen	0.5 − 1.0
Fat	1.0 − 2.0
Blood	0.17 − 0.24
Plasma	0.01
Bone	16.0 − 23.0

Approximate attenuation values for tissue penetrated for human tissue

Pulse-echo: depth of 10 cm, $f_0 = 5$ MHz, attenuation 0.7 dB/[MHz·cm]:

Attenuation: $\mathbf{2} \cdot 10 \cdot 5 \cdot 0.7 = 70$ dB

Amplitude attenuation transfer function for a plane wave

$$|H(f;z)| = \exp(-(\beta_0 z + \beta_1 f z)), \quad (1)$$

z - depth in tissue
f - frequency
β_0 - frequency-independent attenuation
β_1 - frequency-dependent term expressed in Np/[MHz·cm] (Np - Nepers)

Converted to Nepers by dividing with 8.6859

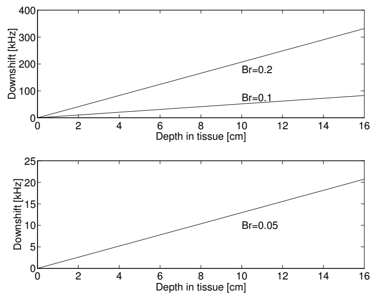
0.7 dB/[MHz·cm] is 0.0806 Np/[MHz·cm].

Frequency dependent term is the major source of attenuation

Frequency independent term is often left out.

Effect of attenuation

Down-shift in center frequency: $f_{mean} = f_0 - (\beta_1 B_r^2 f_0^2) z$



f_0	Center frequency of transducer
c	Speed of sound
v	Blood velocity
β_1	Frequency dependent attenuation
B_r	Relative bandwidth of pulse

$f_0 = 3$ MHz, $\beta_1 = 0.5$ dB/[MHz·cm]

FDA safety limits

	$I_{spta.3}$ mW/cm^2		$I_{sppa.3}$ W/cm^2		I_m W/cm^2		MI
Use	In Situ	Water	In Situ	Water	In Situ	Water	
Cardiac	430	730	190	240	160	600	1.90
Peripheral vessel	720	1500	190	240	160	600	1.90
Ophthalmic	17	68	28	110	50	200	0.23
Fetal imaging (a)	94	170	190	240	160	600	1.90

Highest known acoustic field emissions for commercial scanners as stated by the United States FDA (The use marked (a) also includes intensities for abdominal, intra-operative, pediatric, and small organ (breast, thyroid, testes, neonatal cephalic, and adult cephalic) scanning) from the the September 9, 2008, FDA Guidance.

Discussion from last

Use the ultrasound system from the first discussion assignment

1. What is the largest pressure tolerated for cardiac imaging?

2. What displacement will this give rise to in tissue?

Discussion assignment from last time

B-mode system, 10 cm penetration, 300 λ

Frequency 4.5 MHz, one cycle emission.

Plane wave with an intensity of 730 mW/cm^2 (cardiac, water)

$z = 1.48 \cdot 10^6$ kg/[m^2 s]

Pulse emitted: $f_0 = 4.5$ MHz, $M = 1$ period, $f_{prf} = 7.5$ kHz, $T_{prf} = 133\mu$s

Pressure emitted:

$$
\begin{aligned}
I_{spta} &= \frac{1}{T_{prf}} \int_0^{M/f_0} I_i(t, \vec{r})dt = \frac{M/f_0}{T_{prf}} \frac{P_0^2}{2z} \\
p_0 &= \sqrt{\frac{I_{spta} 2z T_{prf}}{M/f_0}} = \sqrt{\frac{0.730 \cdot 10^4 \cdot 2 \cdot 1.48 \cdot 10^6 \cdot 133 \cdot 10^{-6}}{1/4.5 \cdot 10^6}} = 3.60 \cdot 10^6 \text{ Pa} = 36.0 \text{ atm}
\end{aligned}
$$

The normal atmospheric pressure is 100 kPa.

Example continued

The particle velocity is $U_0 = \frac{p_0}{Z} = \frac{3.60 \cdot 10^6}{1.48 \cdot 10^6} = 2.43$ m/s.

The particle velocity is the derivative of the particle displacement, so

$$z(t) = \int U_0 \sin(\omega_0 t - kz)dt = \frac{U_0}{\omega_0} \cos(\omega_0 t - kz).$$

The displacement is

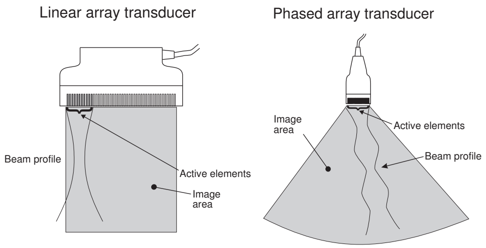
$$z_0 = U_0/\omega_0 = 2.43/(2\pi \cdot 4.5 \cdot 10^6) = 86 \text{ nm}.$$

How do we use ultrasound arrays?

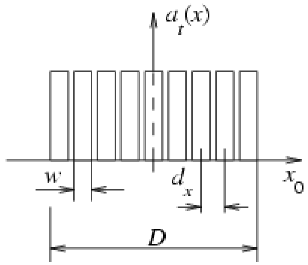
Ultrasound imaging using arrays

- Multi-element transducer arrays are used:
  - Linear arrays - element width $\approx \lambda$
  - Phased arrays - element width $\approx \lambda/2$

1. Focused transmit by applying different delays on elements
2. Field propagates along beam direction. Echoes scattered back.
3. Received signals are delayed and summed (beamformed). Delays change as a function of time (dynamic delays).
4. The same process is repeated for another image direction.



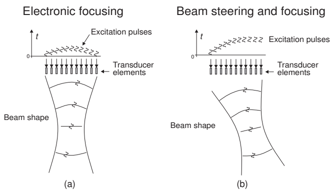
Array geometry



- d_x - Element pitch. For linear array: $\approx \lambda = c/f_0$, for phased array: $\approx \lambda/2$
- w - Width of element
- $k_e = d_x - w$ - Kerf (gap between elements)
- $D = (N_e - 1)d_x + w$ - Size of transducer

- Commercial 7 MHz linear array:
  - Elements: $N_e = 192$, 64 active at the same time
  - $\lambda = c/f_0 = 1540/7 \cdot 10^6 = 0.22$ mm
  - Pitch: $d_x = 0.208$ mm
  - Width: $D = 3.9$ cm
  - Height: $h = 4.5$ mm
  - Kerf: $k_e = 0.035$ mm

Focusing and beamforming



Time from the center of element to field point:

$$t_i = \frac{1}{c}\sqrt{|\vec{r}_i - \vec{r}_f|^2}$$

$\vec{r}_f$ - position of the focal point
$\vec{r}_i$ - center of physical element number i

Reference point on aperture (defines time $t = 0$):

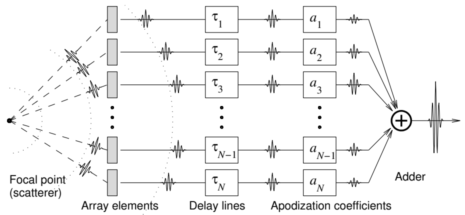
$$t_c = \frac{1}{c}\sqrt{|\vec{r}_c - \vec{r}_f|^2}$$

$\vec{r}_c$ - reference center point on the aperture.

Delay to use on each element of the array:

$$\Delta t_i = \frac{1}{c}\left(\sqrt{|\vec{r}_c - \vec{r}_f|^2} - \sqrt{|\vec{r}_i - \vec{r}_f|^2}\right)$$

Beamforming in Modern Scanners



$$s(t) = \sum_{1}^{N_{xdc}} a_i y_i(t - \tau_i)$$

$$\tau_i = \frac{|\vec{r}_c - \vec{r}_f| - |\vec{r}_i - \vec{r}_f|}{c}$$

- a_i - Weighting coefficient (apodization)
- $y_i(t)$ - Received signal
- $\vec{r} = [x, y, z]^T$ - Spatial position
- $\vec{r}_i$ - Position of transducer element,
- $\vec{r}_c$ - Beam reference point
- $\vec{r}_f$ - Focal point
- c - Speed of sound

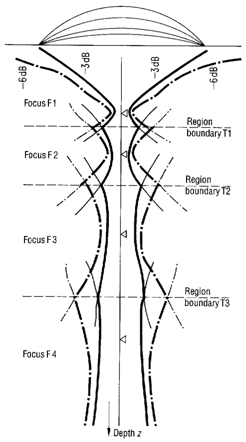
Focusing and apodization time lines

Focusing:	From time	Focus at
	0	$\vec{r}_1$
	t_1	$\vec{r}_1$
	t_2	$\vec{r}_2$
	$\vdots$	$\vdots$

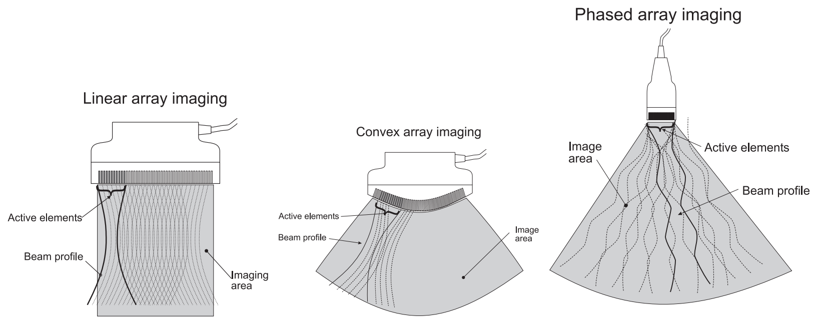
Purpose: to maintain a roughly constant F-number $F_{\#} = \frac{z}{D}$

Lateral resolution at focus $\approx \lambda F_{\#}$

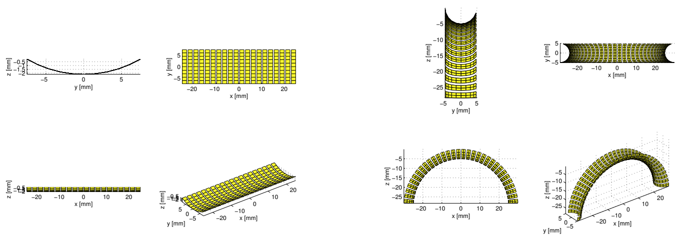
Apodization:	From time	Apodize with
	0	$a_{1,1}, a_{1,2}, \cdots a_{1,N_e}$
	t_1	$a_{1,1}, a_{1,2}, \cdots a_{1,N_e}$
	t_2	$a_{2,1}, a_{2,2}, \cdots a_{2,N_e}$
	t_3	$a_{3,1}, a_{3,2}, \cdots a_{3,N_e}$
	$\vdots$	$\vdots$



Imaging methods

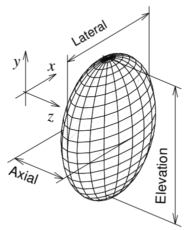


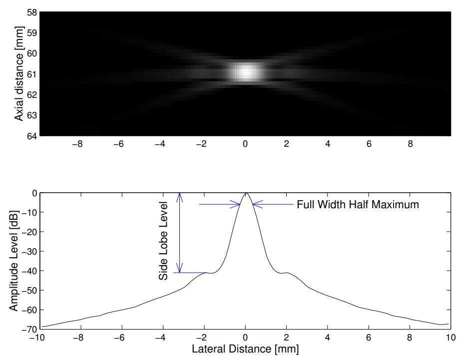
Typical transducers



Linear or phased array Convex array (curvature exaggerated)

PSF Characteristics





- The PSF is three dimensional
- The B-mode images are only 2-D
- Displayed on a logarithmic scale
- Maximum taken along z
- Parameters used: FWHM, side- and grating-lobe level

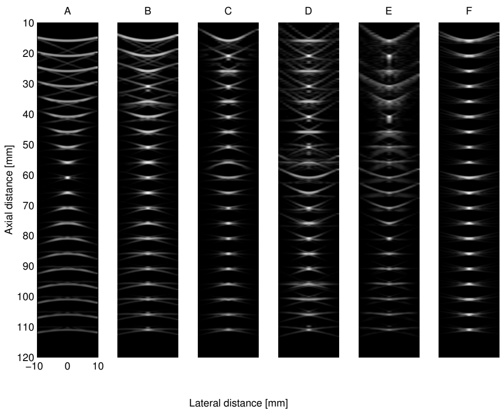
System Characterization

A system can be characterized by the *point-spread-function* (PSF). The point spread function is:

$$\mathbf{p}(\vec{x}, t) = v_{pe}(t) \underset{t}{*} h_{pe}(\vec{r}, t).$$

Examples of PSF without apodization:

- A - one focal zone
- B - 6 receive focal zones
- C - 6 xmt and rcv zones
- D - 128 elem, 4 xmt zones, 7 rcv zones
- E - 128 elem, 4 xmt zones, dynamic rcv
- F - 128 elem, xmt $F_{\#} = 4$, rcv $F_{\#} = 2$

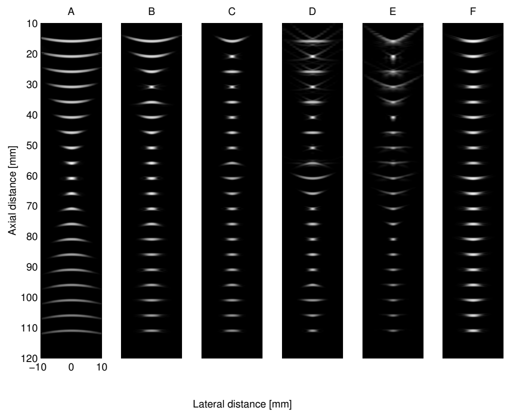


Sidelobe Reduction

The sidelobes can be improved by applying apodization.

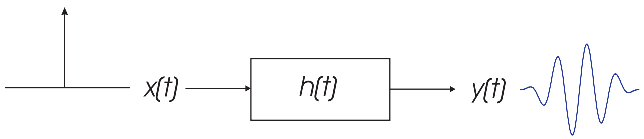
Examples of PSF with apodization:

- A - one focal zone
- B - 6 receive focal zones
- C - 6 xmt and rcv zones
- D - 128 elem, 4 xmt zones, 7 rcv zones
- E - 128 elem, 4 xmt zones, dynamic rcv
- F - 128 elem, xmt $F_{\#} = 4$, rcv $F_{\#} = 2$



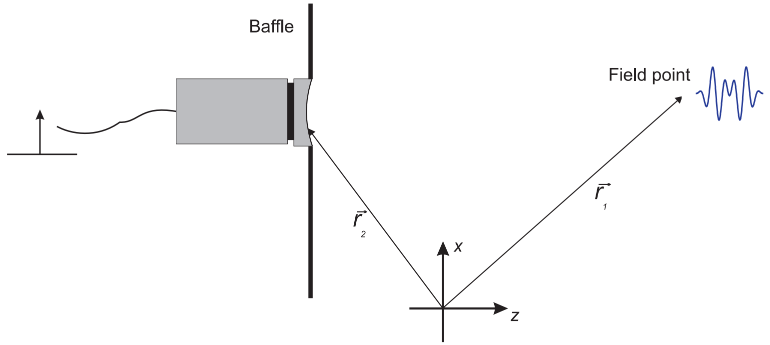
How can we calculate the ultrasound fields?

Linear Electrical System



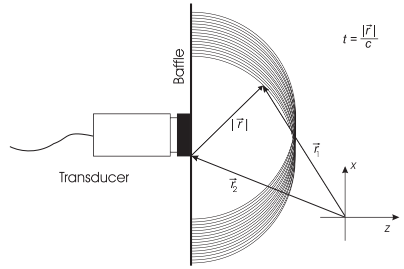
Fully characterized by it's impulse response

Linear Acoustic System



Impulse response at a point in space - Spatial Impulse Responses - $h(\vec{r}, t)$

Huygens' Principle



Arrival times: $t = d/c$, summation of spherical waves

Moving the point results in a new impulse response:

Spatial Impulse Responses - h

Rayleigh's Integral

$$p(\vec{r}_1, t) = \frac{\rho_0}{2\pi} \int_S \frac{\dfrac{\partial v_n(\vec{r}_2, t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{\partial t}}{|\vec{r}_1 - \vec{r}_2|} d^2\vec{r}_2$$

$|\vec{r}_1 - \vec{r}_2|$ - Distance to field point
$v_n(\vec{r}_2, t)$ - Normal velocity of transducer surface

Summation of spherical waves from each point on the aperture surface

Spatial impulse response:

$$h(\vec{r}_1, t) = \int_S \frac{\delta(t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{2\pi \, |\vec{r}_1 - \vec{r}_2|} dS$$

Emitted field:

$$p(\vec{r}_1, t) = \rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1, t)$$

Pulse echo field:

$$v_r(\vec{r}_1, t) = v_{pe}(t) * h_{pe}(\vec{r}_1, t) = v_{pe}(t) * h_t(\vec{r}_1, t) * h_r(\vec{r}_1, t)$$

Ultrasound fields

Emitted field:

$$p(\vec{r}_1, t) = \rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1, t)$$

Pulse echo field:

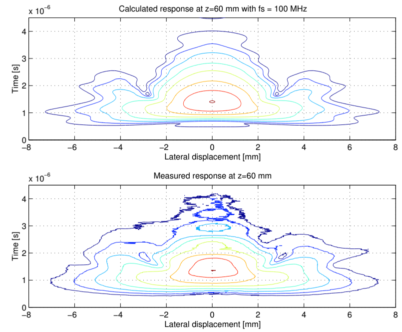
$$\begin{aligned} v_r(\vec{r}_1, t) &= v_{pe}(t) * f_m(\vec{r}_1) * h_{pe}(\vec{r}_1, t) \\ &= v_{pe}(t) * f_m(\vec{r}_1) * h_t(\vec{r}_1, t) * h_r(\vec{r}_1, t) \\ f_m(\vec{r}_1) &= \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c} \end{aligned}$$

Continuous wave fields:

$$\mathcal{F}\{p(\vec{r}_1, t)\}, \qquad \mathcal{F}\{v_r(\vec{r}_1, t)\}$$

All fields can be derived from the spatial impulse response.

Point spread functions



Point spread function for concave, focused transducer
Top: simulation top
Bottom: tank measurement (6 dB contour lines)

Discussion for next time

What are the focusing delays on an array.

Parameters: 64 element array, 5 MHz center frequency, λ pitch, all elements used in transmit

Focusing is performed directly down at the array center.

1. Imaging depth of 1 cm: How much should the center element be delayed?

2. Imaging depth of 10 cm: How much should the center element be delayed?

Learned today

- Focusing of arrays using delay-and-sum beamforming

- Point spread functions

- Calculation of fields using spatial impulse response

Next time: Ch. 2 in JAJ, pages 36-44 on array geometries and their design